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COLLEGE OF ENGINEERING
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Final Report

THE MECHANICS OF WALKING VEHICLES

A Feasibility Study

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A = acceleration, fps^2

A^x = $\ddot{x}$ = component of acceleration in x direction, fps^2

C = circumference, ft or in.

F = force, lb

F^x = component of force in x direction, lb

I = mass moment of inertia, $lb\text{-}sec^2\text{-}ft$

K = a constant

M = moment of a force, ft-lb

m = mass, $lb\text{-}sec^2/ft$

r = radius or length, ft or in.

P = an external force, lb

R = reaction force, lb

T = torque, ft-lb

t = time, sec

V = velocity, fps

V^x = $\dot{x}$ = component of velocity in x direction, fps

x = distance along a horizontal axis, ft

y = distance along a vertical axis, ft

θ = the angle between a link or vector and a reference axis

$\dot{\theta}$ = ω = angular velocity, rad/sec

$\ddot{\theta}$ = α = angular acceleration, rad/sec^2

τ = period, or duration of a cycle, sec

ACKNOWLEDGMENT

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ABSTRACT

Object. It is well known that animals can travel over rough terrain at speeds much greater than those possible with wheeled or tracked vehicles. Even a human being, by "getting down on all fours" if necessary, can travel or climb over terrain which is impossible for a wheeled or tracked vehicle. Nature, apparently, has no use for the wheel. It is therefore of considerable interest to learn what machines for land locomotion can do if they are designed to imitate nature.

Accordingly, it is the purpose of this investigation to initiate the science of walking and crawling machines. Specifically, to define the terminology, discover the laws of walking, and find what manner of mechanisms can be useful. Another purpose is to study the feet of walking vehicles and to learn what requirements govern their shape and action. It will also be exceedingly useful to discover the speeds with which walking vehicles can operate, and how these speeds compare with wheeled and tracked vehicle speeds. Finally, all vehicles must be controlled—by a human or other means. Therefore it is necessary to investigate methods of controlling such vehicles.

Summary. A class of walking mechanisms driven by a rotating crank has been defined. Because the inertia forces cannot be balanced, these machines are only useful for very slow speeds, say less than 5 mph. It is possible to design them, however, so that the feet are like long runners or skis which would be useful for travel over snow or soft ground.

Another class of mechanisms, driven by hydraulic means, has been developed which is capable of traveling over rough terrain at speeds somewhat greater than that for wheeled or tracked vehicles.

The laws governing the design of walking machines, including the feet and shoes, have been defined, and methods presented for balancing the inertia forces. The control means for the rotating-crank mechanisms is found not to be too satisfactory, but the hydraulically operated mechanisms have great versatility in control.

Finally, a means is suggested for further investigation which promises to give very high speeds for hydraulic vehicles over rough terrain.

Recommendations. A combined research and development program is suggested. The purpose of the development program is the design, and finally construction, of an experimental vehicle. This involves defining materials, parts, and sizes in relation to the vehicle size, weight, and speed. In particular, the preliminary design work would be concerned with developing the hardware for the feet and legs, suspending these from the vehicle, and then calculating the actual forces that would be involved.

Research should be aimed at defining the control system in precise terms and then analyzing its performance and finally that of the entire vehicle. Research should also be undertaken into the possibility of developing a high-speed vehicle.

1. INTRODUCTION

1-1. DEFINITIONS AND NOMENCLATURE

This investigation has for its purpose a study of the feasibility of utilizing walking, or crawling, machines for land locomotion. In particular, the study is aimed at discovering some of the capabilities and limitations of walking vehicles as well as the general configuration, control system, and power-supply requirements and characteristics of such vehicles.

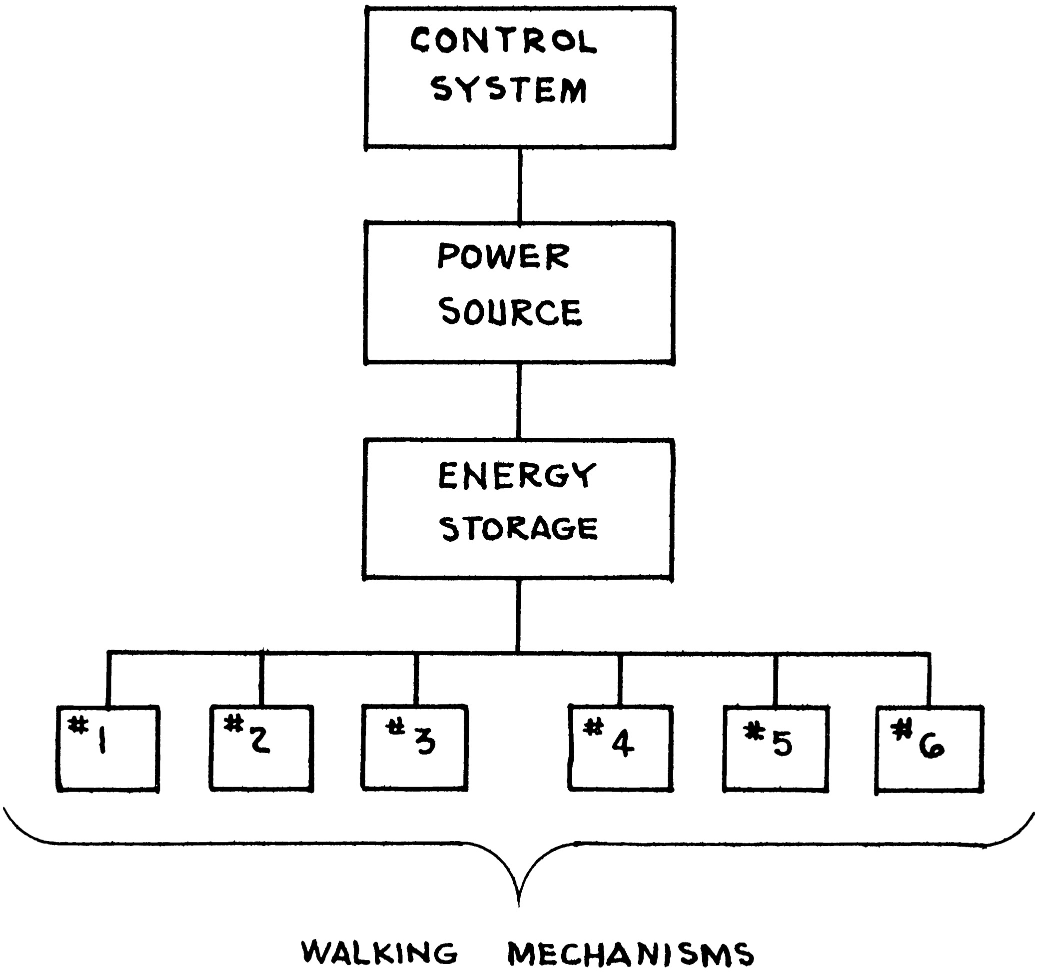


Fig. 1-1. Block diagram of a walking vehicle.

The elements of a walking vehicle are illustrated by the block diagram of Fig. 1-1. The power source is used to overcome friction between the feet of the vehicle and the terrain, to make up for frictional losses which occur in the moving parts, and to lift the vehicle over obstructions and up grades. The control system provides a means of controlling the speed, direction, length of stride, height of step, and elevation of vehicle above the mean terrain level. The legs or the unit means of locomotion are defined as walking mechanisms, in this report, and are identified by the numbered blocks in Fig. 1-1. The energy-storage device serves to level the power requirements by providing energy for acceleration of some of the walking mechanisms and absorbing the energy released by the deceleration of other mechanisms.

1-2. REQUIREMENTS OF A WALKING MECHANISM

An ideal walking mechanism is one which fulfills the following requirements:

1. It must have a uniform velocity while the feet are in contact with the terrain.

2. The stride must be long in relation to the physical dimensions of the walking mechanism.

3. The length of the stride must be controllable by the vehicle operator.

4. The height of the step (return stroke of foot) should be large compared with the dimensions of the walking mechanism.

5. The height of the step should be controllable by the vehicle operator.

6. It should have a high stride-time to return-time ratio. This means that the foot should be in contact with the terrain during a major portion of a cycle of operation.

7. The speed of a walking mechanism should be capable of variation independently of other walking mechanisms on the same vehicle. This requirement provides the means for direction control.

8. The walking mechanism should be such that it can move the vehicle in either the forward or the backward direction.

9. The inertia forces and inertia torques acting on the vehicle should be balanced.

10. The energy required to lift the foot at the end of stride and return the foot to the beginning of stride should be recoverable.

11. The height of the body of the vehicle above a mean terrain level should be capable of control by the operator.

These requirements are used in this report as a means of qualitatively judging, or rating, the various proposed walking mechanisms. Obviously a small walking mechanism is superior to a large one, other things being equal; similarly, a mechanism composed of a few relatively simple parts is much superior to one that is quite complicated consisting of many elements. So these requirements represent something that it is hoped can be achieved.

1-3. SCOPE OF THE INVESTIGATION

By far the major effort of this investigation was concentrated on mechanically operated and hydraulically operated walking mechanisms. The requirements of uniform velocity (requirement No. 1) and of recoverable energy (No. 10) were used from the very beginning to accept or to reject proposed walking mechanisms, and, unless a means first satisfied both of these requirements, it was not investigated. This criterion thus ruled out pneumatic means since none were found which satisfied the initial requirements. Solutions using electrical actuation were not investigated.

Because of the fact that the requirement of recoverable energy (requirement No. 10) was used as an initial requirement, the efficiencies of the various means which were investigated are not of importance in evaluating the relative merits of the mechanisms. The word "efficiency" is sometimes loosely used by nontechnical people to describe the relative simplicity or complexity of a machine. It is true that a machine having only a few simple parts is less expensive to construct than one having many complicated parts, but this has nothing to do with the efficiencies of the two machines. In this report the word efficiency will seldom be employed. When it is used it will be employed to designate only the ratio of the useful output of a machine to the work or energy put in.

A study of foot motion, that is, the path of and characteristics of the motion of the foot relative to the terrain, is necessary in order that the geometry and the laws governing the action of the foot can be defined. The results of such a study are included in this report. Also included are the laws governing the number of legs and their arrangement.

A walking machine should be capable of going around or over and down hills, across ditches, under bridges, and around obstructions. Consequently it is necessary to control the speed and direction of motion of such vehicles. It is also necessary to control the length and height of the foot motion. For this reason an investigation of the control systems is a necessary part of this report.

Under the assumption that walking vehicles will have unique advantages over wheeled vehicles, a group of recommendations for further study and investigation have been included. An expression of these advantages has not been included because this would necessarily entail a study of wheeled vehicles for proper comparison. Such a comparison is therefore left to others.

2. MECHANICS OF THE FOOT

2-1. LOCUS RELATIVE TO VEHICLE

A useful means of obtaining a preliminary estimate of the value of proposed walking mechanisms is to graphically generate the closed curve, called the locus, which contains all positions of a given point on the foot. In generating this curve the vehicle is imagined to be stationary, and the terrain or roadway to be moving backward at the vehicle velocity.

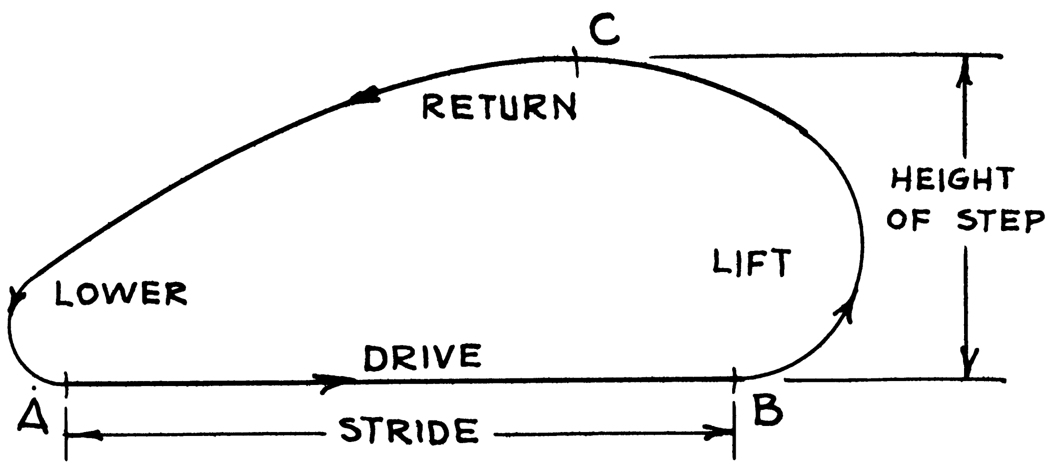


Fig. 2-1. Locus of the foot.

Figure 2-1 illustrates a typical locus. The distance AB is the stride; this line should be straight and a point traversing this portion of the locus should move at a constant velocity. The remaining portion of the curve BCA constitutes the lift, return, and lower phases of the cycle. The shape of the lift and lower portions of the curve is useful in determining the action of the foot in initial and final contact with the terrain. The shape of the return portion of the curve is immaterial except as it affects the peak acceleration of the cycle.

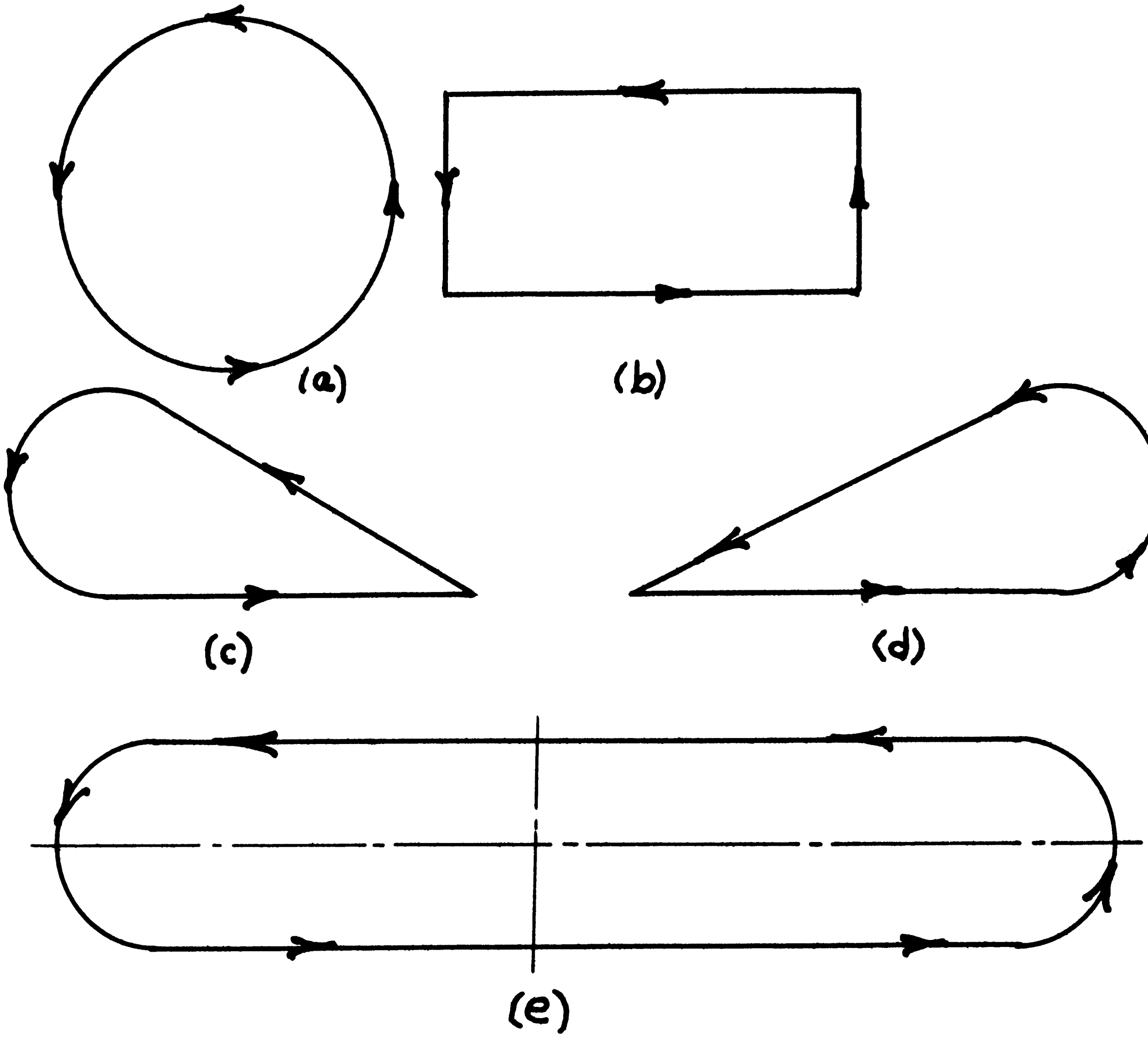


Fig. 2-2. Foot loci.

Figure 2-2 illustrates a group of paths. A point traveling on the circle of Fig. 2-2a with uniform velocity would have a minimum acceleration, but the length of stride would be infinitely small. In fact this illustrates that a wheel may be classified as a special case of a walking mechanism.

The rectangular locus of b offers distinct advantages. It gets the foot off the ground quickly and in the most direct manner at the end of stride and replaces the foot in the same manner at the beginning of stride. But time is required for the lower phase of the action as well as for the drive phase. If the foot should encounter the roadway before the lower phase is completed the vehicle would either pause for the remaining interval, or sliding would occur. Furthermore where velocities are at all appreciable a locus having corners is abhored by nature. The result would be infinitely large accelerations resulting in unsatisfactory performance and life.

The loci of Fig. 2-2c and d represent paths which can be generated by four-bar linkages. In some cases a loop exists at the pointed end. Unsatisfactory performance may be expected from both of these because of the high accelerations at the pointed ends, and because of the manner in which the foot contacts or finishes contact with the roadway at the pointed ends.

This discussion is intended to demonstrate that the locus of Fig. 2-2e is an ideal one. It has a long, straight, stride. The return portion of the locus is direct, and promises to give minimum accelerations. The lift and lower phases of the action resemble that of a wheel and should have acceleration characteristics not too different from a wheel. Furthermore, if the locus is symmetrical about both horizontal and vertical center lines, then the possibility exists of getting balanced inertia forces by properly phasing a number of walking mechanisms on like paths. If the path is symmetrical then vehicle reversibility is no problem. And finally, the shape of the lift and lower phases seems to be the best compromise between the circular locus of a and the rectangular one of b.

2-2. VELOCITY AND MOTION CONSIDERATIONS

In the case of a wheeled vehicle the tread of the wheel contacts all particles of the roadway along the path of travel. But the shoe of a walking vehicle contacts only spots or small areas along the path of travel. The next spot of contact may be at a higher, a lower, or at the same relative elevation, as the preceding one. Consequently it is necessary to investigate the action of the shoe during the lower and lift phases as well as during the stride phase of the action.

Another factor of importance concerns the foot, or leg, or member to which the shoe is attached. Just as the leg of a horse or a human being assumes a number of attitudes relative to the ground during stride, so also does the leg of a walking machine. In all cases investigated the leg had a rocking motion relative to the terrain during stride. The angle of this rock and the relative location of this angle varies considerably with different mechanisms. In some mechanisms the rock is with the direction of travel; in others it is against the direction of travel; and there are even some mechanisms in which the leg rocks in both directions during stride. The magnitude of this angle affects the size of the shoe and the variation of the leg forces. The relative location of the rock angle influences the magnitude of the leg forces.

The meaning of rock angle and pressure angle as used in this report is illustrated in Fig. 2-3. As shown, the pressure angle varies during stride, and the rock angle is simply the difference between the maximum and minimum pressure angles.

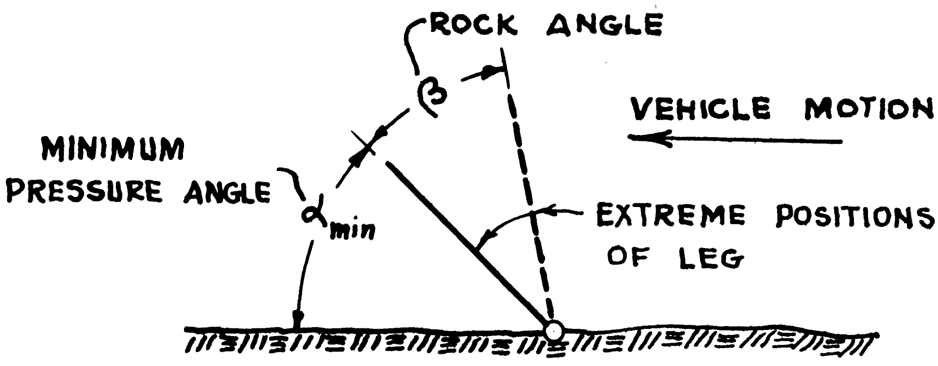


Fig. 2-3. Definition of rock angle and pressure angle.

One might specify that the difference between walking and crawling is essentially in the height of step used. Thus a machine can be said to "walk" when the height of step is large, and to "crawl" when the step height is small. Having made this distinction one might reason from Fig. 2-3 that crawling machines should have low pressure angles and low rock angles, and that such machines would be useful for climbing relatively smooth steep grades. On the other hand it is doubtful if a constantly small pressure angle can be associated with a "high-stepping" machine, and consequently, walking machines should be most useful over rough terrain of moderate grade. All of these conclusions are based primarily on the leg forces. Thus we would prefer a leg which is always in pure compression. Later we shall discover that this is seldom true and that bending forces exist in the legs because of the static weight of the vehicle and because of the acceleration forces.

One can write the relative velocity equation in the form

$$V_{FT} = V_{VT} + V_{FV} \tag{2-1}$$

which states that the velocity of the foot relative to the terrain equals the velocity of the vehicle relative to the terrain plus the velocity of the foot relative to the vehicle. Equation (2-1) is a vector relationship. During stride $V_{FT} = 0$ and we have

$$V_{FV} = - V_{VT}$$

which means that the foot moves backwards relative to the vehicle but at the same velocity as the vehicle. It is interesting to plot a graph showing the relationship of the three terms of Eq. (2-1) for all phases of a cycle. Assuming equal times for the four events and a linear velocity change for the lift and lower phases, we get the graph of Fig. 2-4. Examining this

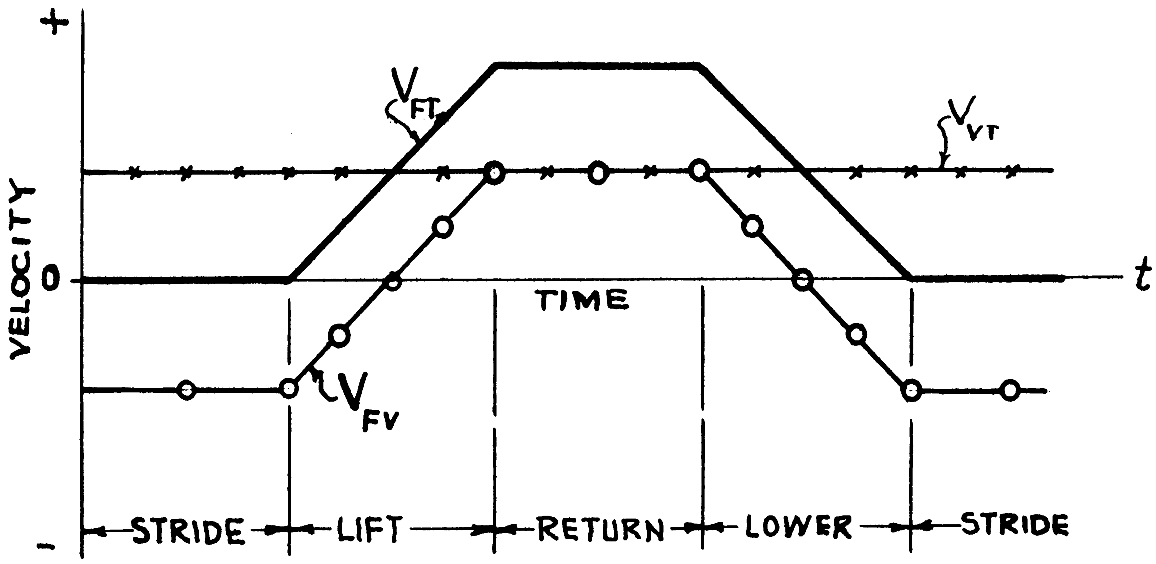


Fig. 2-4. Relative velocity diagram.

graph we see that the velocity of the foot relative to the terrain V_{FT} is zero during stride, as it should be. During the return stroke, however, the foot is moving twice as fast as the vehicle and in the same direction. Note that, in the first half of lift and the last half of lower, the foot is moving slower than the vehicle. This means, during lift, if the foot is still in contact with the terrain, or if it meets an obstruction and comes into contact again, that the vehicle will pull or drag the shoe over the terrain. Similarly, during the last half of lower, relative motion between the shoe and the terrain will occur if contact is made before the beginning of stride. Relative motion between the shoe and the terrain is much more likely to occur on lower than on lift. Of course, if the vehicle operator has control over the length of stride and the height of step, then the possibility exists for him to step or walk over short obstructions and thus avoid sliding of one shoe.

It is interesting to contrast the operation of a wheel when meeting an obstruction with that of the foot of a walking machine upon meeting the same obstruction. Both cases are shown in Fig. 2-5. In a a wheeled vehicle moving with a velocity V meets with an obstruction. Point A is the point of contact and the periphery of the wheel has the same velocity V but its horizontal component V^h is less than V and consequently the vehicle instantaneously (except for the elasticity of the tire) tries to change its velocity to V^h causing shock to the vehicle. In b the walking

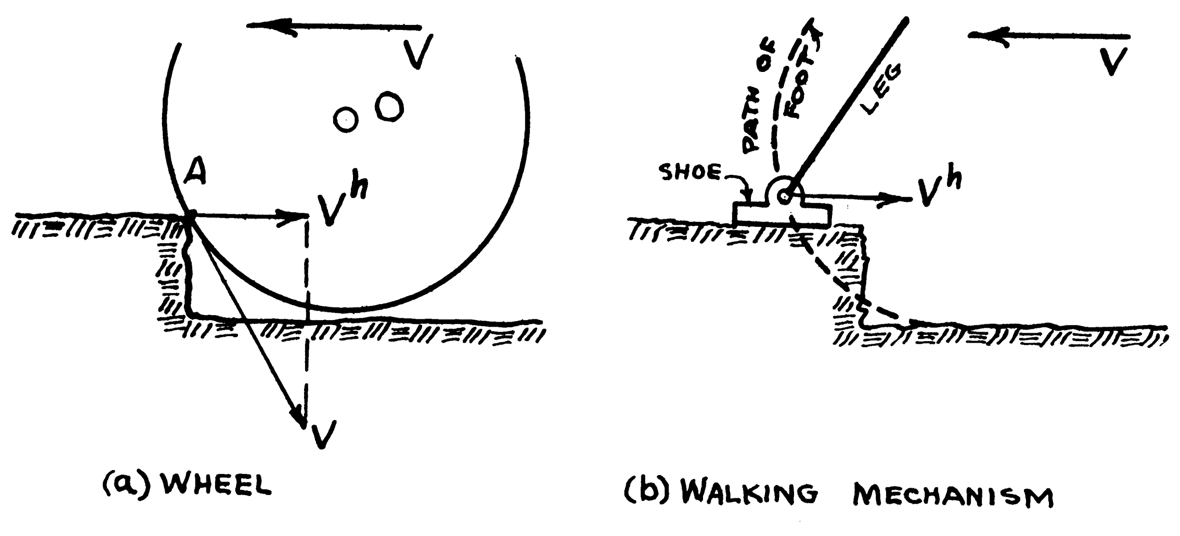


Fig. 2-5. Comparison of a wheel and a foot.

machine meets the same obstruction. The velocity of the foot V^h also differs from the velocity of the vehicle V so that the same conditions prevail velocitywise. The wheel is not capable of sliding until it has overcome the obstruction. Sometimes the foot will not slide either, but, in general, it seems that some sliding will generally take place and ease the shock. Though it is impossible to quantitatively evaluate the differences except by actual experimentation, it would appear that properly designed shoes and feet will permit a moderate increase in speed for the walking vehicle.

2-3. THE FEET AND SHOES

The basis for the design of the feet and shoes of a walking vehicle has been established in the preceding sections. It is clear that shoes having claws or spikes rigidly attached to the legs would constitute a most unsatisfactory solution. Penetration into the earth of such members would prevent relative motion and greatly increase the shock forces on the vehicle. On the other hand, if the relative motion always occurs between the shoe and the foot, or in the foot itself, then there is no reason why spikes or claws could not be used if the nature of the terrain justifies them.

Later in this investigation we shall find that wide shoes, in many cases, greatly increases the width of the vehicle. The effort should therefore be concentrated in the direction of obtaining relatively narrow shoes. This now means that we have the following list of desirable characteristics to guide us in designing the shoes:

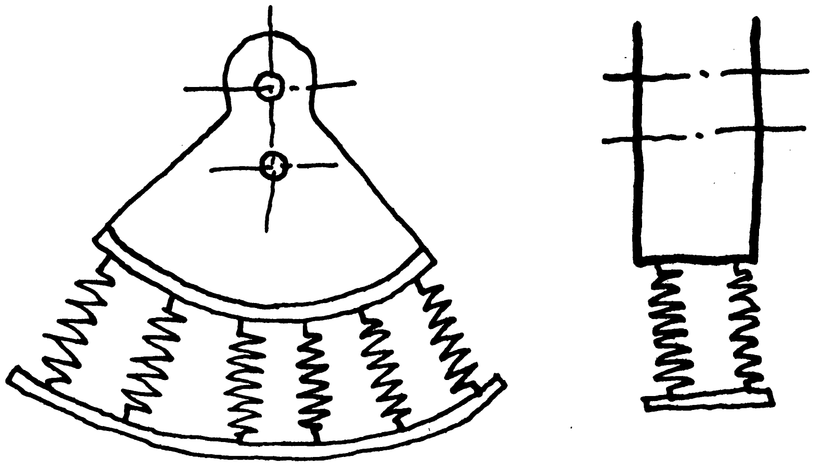
1. They should be narrow.

2. They should be capable of supporting large shear deflections in the direction of travel. Note that this will greatly reduce, or even eliminate, the amount of sliding between the shoe and the terrain.

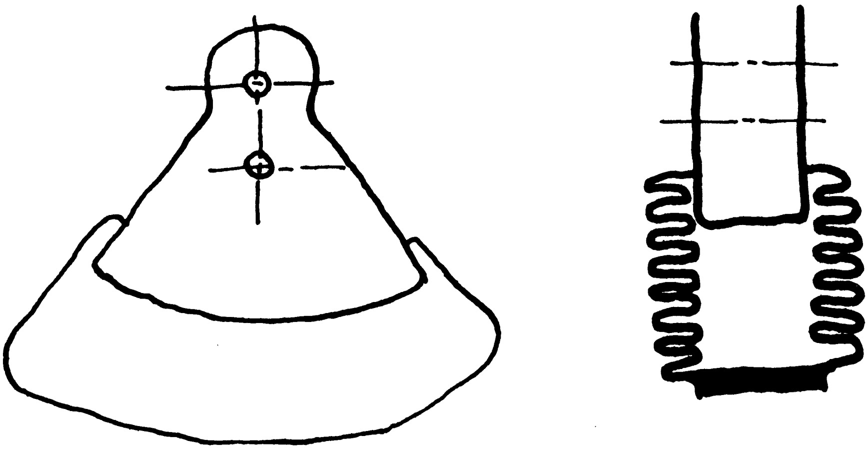
3. They must accommodate the rock angle of the leg.

4. They should be capable of supporting and absorbing shock in the vertical direction.

These requirements may be difficult to satisfy. Two proposals, to indicate the general line along which thinking should be encouraged, are illustrated in Fig. 2-6. The shoe mounted on helical springs satisfies



(a)



(b)

Fig. 2-6. (a) Shoe on helical springs; (b) shoe is an inflated rubber tire, bellows shaped for greater shear deflection.

requirements No. 2 and No. 4 extremely well, but it may have a short life because of fatigue failure of the springs. There is nothing new about using an inflated rubber tire as shown in Fig. 2-6b. Some arrangement, such as the bellows-shaped sides, must be included so as to cause the tire to support shear deflections.

2-4. SKIS OR RUNNERS

When a pair of legs, on a walking machine, are separated by a reasonable distance and synchronized, then their lower extremities may be joined by runners or skis in order to provide a very long narrow shoe. Such an arrangement should be especially useful for walking over soft terrain such as snow, swampy ground, or loose sand. The possibility is noted, however, that the use of two legs in synchronism by its very nature prevents the phasing of one leg relative to another in order to balance inertial effects.

2-5. BALANCING

In this section we shall try to consider in a preliminary and somewhat nonrigorous manner some of the problems involved in balancing the inertia forces and torques of a walking vehicle. These are closely related to the shape of the locus of the foot and so such a study should be useful later in forecasting the probable success of various proposed solutions.

Figure 2-7 illustrates an equivalent walking mechanism traversing the ideal locus of Sec. 2-1. It is called an equivalent walking mech-

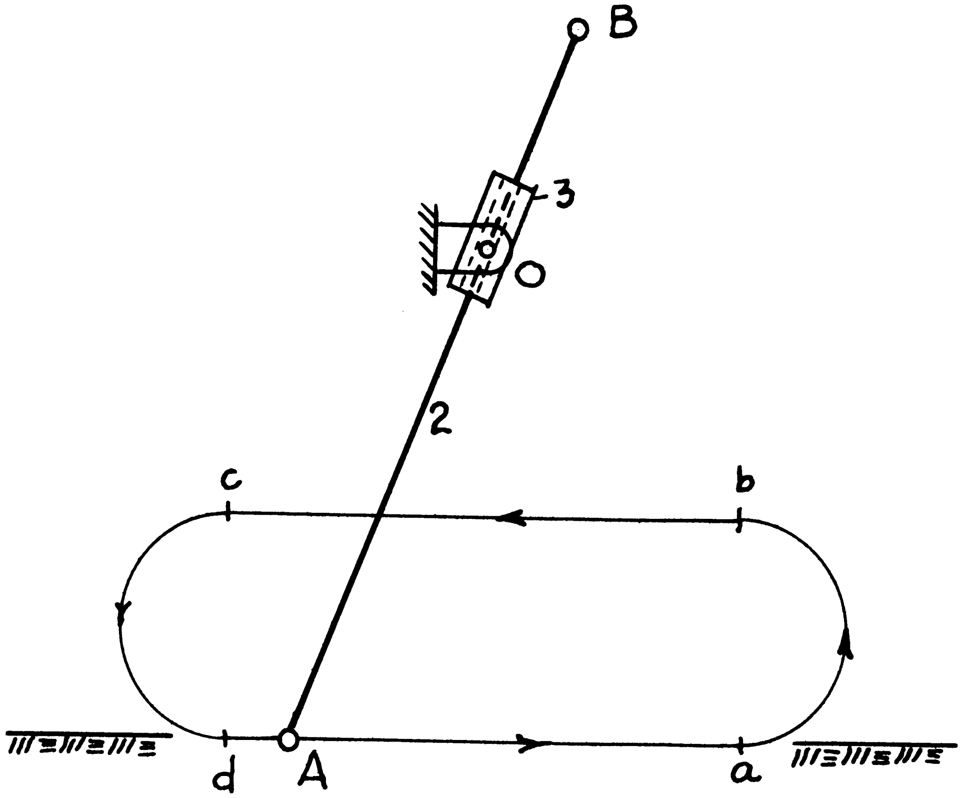


Fig. 2-7. Equivalent mechanism.

anism because we wish it to represent the general class of mechanisms capable of traversing the path abcda. Point A represents the foot and this is the point that generates the locus. Link 2 represents the leg. Link 3 is a rocking block through which link 2 can slide. The driving force must be supplied to point B on the leg, but it is not necessary that we specify the details for the purpose of this discussion.

For convenience let us visualize point A as traversing the locus at a uniform velocity always tangent to the path. Then, because the distance OA is always changing, the leg will always have an angular acceleration. This will be clockwise part of the time and counterclockwise part of the time, but the acceleration will exist, except when it is changing direction, for the entire cycle. In addition to the angular acceleration, a vertical acceleration takes place during the lift and lower phases of the action.

For the problem as described, the vertical acceleration during lift has a sense which is exactly opposite to the vertical acceleration during the lower phase of the action. This means that, if two legs on a vehicle are out of phase by one-half a period, then, during the lift and lower phases of the action, the vertical inertia forces will be exactly opposite in direction. If the masses of the two legs are the same, and if the lines of action of the two inertia forces are coincident, then the resulting inertia force on the vehicle is zero. This explains the meaning of balancing and how the vertical inertia forces can be balanced.

To summarize: The vertical inertia forces on a walking vehicle are balanced if (1) each pair of legs have their action separated in phase by one half a period, (2) the lines of action of the vertical inertia forces are coincident, and (3) the loci of the feet are identical and are symmetrical about horizontal and vertical center lines.

Next, we consider the horizontal components of the inertia forces. If the leg is statically balanced these result in an inertia torque and it is in this torque that our interest lies. It is clear, from Fig. 2-7, that the inertia forces during stride will be opposite in direction to those which occur during the return phase of the action. It is not so clear that their magnitudes will be different. During lift of the leg the mass center changes its position too. Furthermore the angular acceleration of the leg is different in magnitude during return than it is during stride because the foot A is closer to the center of rotation O. Thus a pair of legs separated by a phase of one-half period will only have their inertia torques partly balanced during the stride and return events. We shall see, in later portions of this report, how, by using 16 or more legs, the inertia torque reaction on the entire vehicle can be made very small.

The preceding discussion shows that two legs are not sufficient to support one corner of a vehicle if the ideal locus is used and if the foot traverses this locus with uniform velocity. In fact, four legs per corner separated in phase by one-quarter period would be required for the vehicle of this discussion.

3. MECHANICALLY OPERATED WALKING MECHANISMS

By mechanically operated walking mechanisms we refer specifically to any walking mechanism consisting of mechanical links driven by a rotating power source. This is in contrast to hydraulically operated walking mechanisms, which, of course, are mechanical too; hydraulically operated mechanisms are discussed in a separate chapter.

3-1. OBJECTIVES AND METHODS OF ATTACK

Figure 3-1 is not a solution to the problem but it represents the general configuration of the class of linkages which were investigated, and is presented as an illustration of some of the problems involved.

The linkage of Fig. 3-1 is known as the crank and rocker linkage. The crank or driver is link 2; it rotates about a stationary center at O_2. Link 4 is the rocker or oscillating follower, and it rotates about another stationary center at O_4. Link 3 is called the coupler, and it is connected to the foot at point B. Link 1, the frame of the vehicle, is not shown.

The linkage shown in the figure will not generate the locus. In fact, basically, this is the problem: What are the dimensions of the four links, and the dimensions of the coupler, which will generate the required locus?

A solution similar to Fig. 3-1 has many attractions. A simple rotating source of power is easily obtained. A linkage containing only pin joints is relatively easy to construct, lubricate, and seal; and it will accommodate large forces. It is well known that closed curves similar to this locus can be generated by a point on the coupler of a four-bar linkage. It is not difficult to cause the coupler to generate a line which is very nearly straight. Furthermore, the length of the straight-line portion of the locus can be quite large in comparison with the crank dimensions O_2A. As an additional incentive the possibility exists of finding a linkage in which the straight-line portion of the loop is generated rather slowly while the crank is turning through a

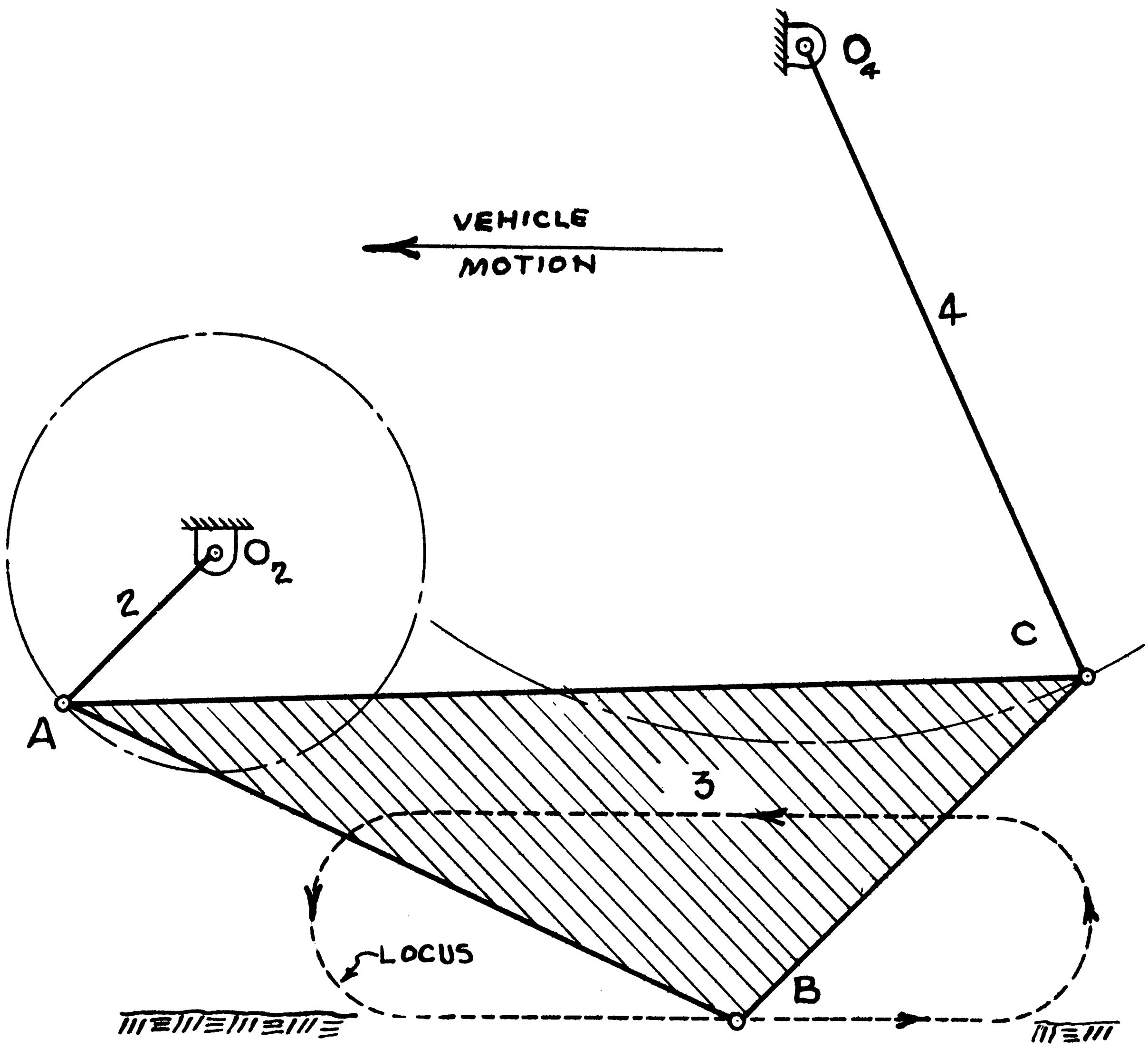


Fig. 3-1. Desired linkage.

major portion (say 60 to 80 per cent) of its total crank angle. This means that the crank may turn through say 270° during stride and use the remaining angle of 90° for the return events of the cycle. Thus, for slow-speed walking machines where inertia forces are not important considerations, the four-bar linkage holds out the promise of giving a vehicle with only a few legs, driven by a simple power source, and having a long stride. Although there are an infinite number of possible solutions to such a problem, the rewards appear to be great. Consequently a great deal of effort was put into the synthesis of such a linkage.

The Hrones-Nelson synthesis[1*] is always a good first approach to such a problem. Since there are an infinite number of solutions however, there are still an infinite number remaining after exhausting this source.

The second method of solution is due to Kurt Hain[2] of the West German Federal Research Institute of Agriculture and is called point-position-reduction. Essentially it involves choosing five points on the path to be generated. By judicious choices of points O_2 and O_4 and the crank length O_2A, points B and C are then located by an inversion process so that point B will pass through the five preselected points.

Freudenstein[3] has programmed Hain's method on the IBM 650 computer so as to obtain a linkage which will generate the locus with the least error. Unfortunately, it is necessary, with Freudenstein's approach, to investigate the problem rather thoroughly using graphical means in advance. In other words Freudenstein's method cannot usually be used to obtain a solution unless one has first been obtained by Hain's graphical method. It is anticipated that more satisfactory means of linkage synthesis by digital computer will become available in the future.

One of the disadvantages of point-position-reduction is that there is no assurance in advance that the resulting linkage will permit the crank to rotate in a complete circle.

3-2. SOLUTIONS

A linkage, representing a group of similar solutions obtained by the Hrones-Nelson method, is shown in Fig. 3-2. The locus generated by point B on the coupler is shown by the dashed lines. The numbers on the locus correspond to the numbered positions of the crank. Thus the stride begins at about station No. 7 and ends at about No. 11. Since the crank rotates 30° between each pair of stations, this corresponds to a crank rotation of 120° for stride. All dimensions are given in terms of a unit crank radius. If the stride is measured, it will be found to be 1.90, which is 1.90 times the radius of the crank.

The distance between stations on the locus gives a good idea of the relative velocities involved. Thus the distances between 7 and 8, 8 and 9, 9 and 10, and 10 and 11, are nearly equal. This means, during stride, uniform angular crank velocity will give nearly uniform vehicle velocity. On the return stroke the distance between stations 2 and 3 is quite large; consequently one can expect a high velocity of the foot

*These numbers refer to references in the bibliography.

during this period. On the other hand, at the two ends of the stroke, the velocity is changing magnitude as well as direction quite rapidly. Therefore we can expect very large accelerations at these ends.

We note also that the return-stroke clearance is only a small fraction of the crank radius, and this is most unsatisfactory. The return-stroke clearance can be improved by moving point B to a new position relative to points A and C, but this always reduces the straight-line portion of the stride.

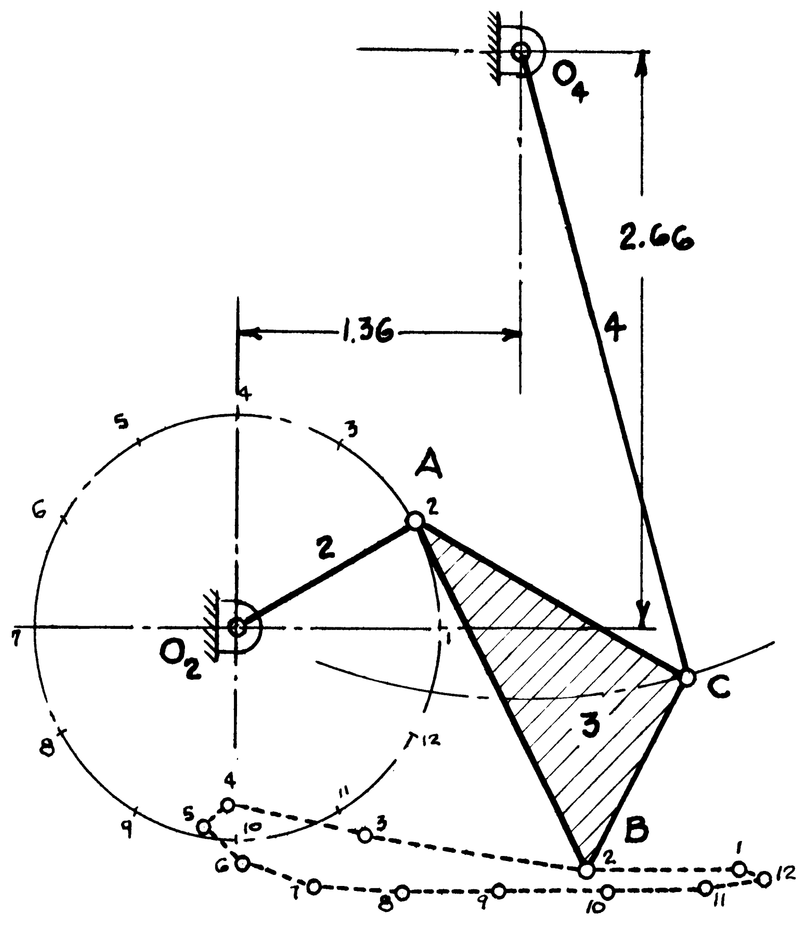


Fig. 3-2. A fair-bar walking linkage.

Another disadvantage of the mechanism of Fig. 3-2 is the fact that the bottom of the crank circle is too close to the roadway surface.

Thorough searching of the Hrones and Nelson figures reveals only a few linkages capable of generating the desired locus. None of these appear to have any advantages over the one of Fig. 3-2. On the other hand if link 4 is permitted to oscillate about a point which is near or below the roadway surface then a great many configurations become available for study. Of course, link 4 cannot actually oscillate about a pivot on the vehicle frame which is located underneath the road; the same effect may be had simply by causing point C to move on the arc of a circle whose center is below the road. Thus in order to investigate this class of linkages we replace link 4 by a slider pivoted at C and constrained to move on a circle arc whose center may be anywhere.

The existence of a slider pivoted on the coupler may not be a satisfactory solution because this requires a cam groove for each leg. Such a groove would be difficult to seal off from mud and dirt on the side of a vehicle and would require rather elaborate bearings and housings. Nevertheless, the desire to obtain something capable of being analyzed (whether it is practical or not), coupled with the complete failure of the point-position-reduction synthesis to yield anything, led to an investigation of four-bar linkages in which the fourth link is a slider operating in a cam groove.

The linkage of Fig. 3-3 is representative, and among the best, of the group utilizing a cam groove. The locus shows a good stride, a fairly good return-stroke clearance, and a lower phase of the action which is not too bad. The lift phase of the action is not good, and the locus is not symmetrical about either the vertical or the horizontal center lines. The fact that the foot velocity during stride is not uniform requires that the crank be driven at a non-uniform velocity. This is a correction means and it can be accomplished by utilizing a pair of non-circular gears. One of these is connected to the crankshaft and the other acts as a driver.

Certain alterations may be made in the mechanism of Fig. 3-3 in order to change the shape of the locus. Lengthening BC increases the return-stroke clearance and causes the stride to become more curved. Increasing the distance AC generally flattens the locus, sharpens the ends, and introduces some curvature into the stride. If the center of curvature of the cam groove is moved further away from the crankshaft, with no other changes, the entire locus is shifted downward, and the length of stride is increased.

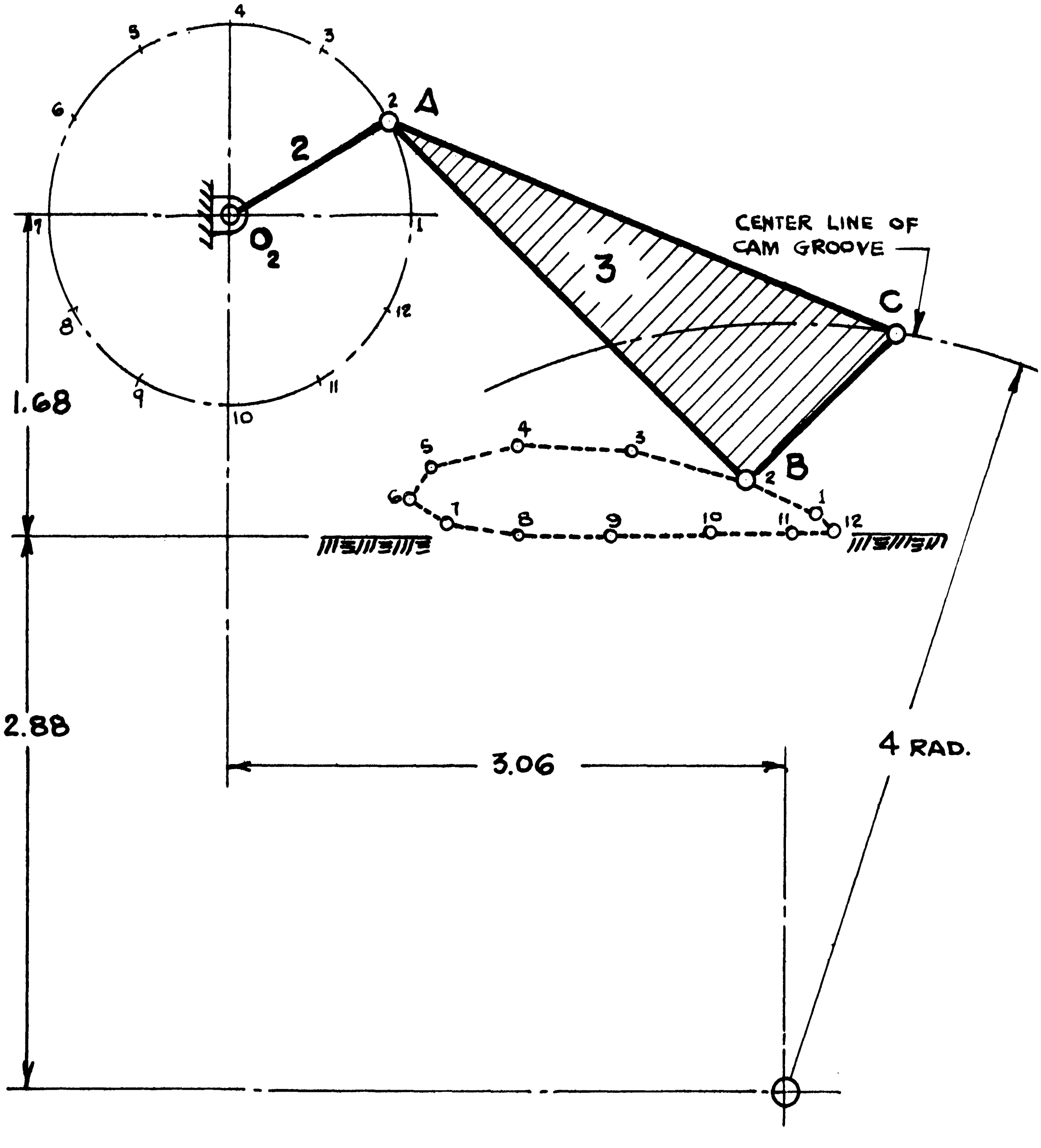


Fig. 3-3. The cam-groove linkage.

Slight changes in the shape of the locus can also be made by altering the curvature of the cam groove. Since there is no reason to make this groove of constant curvature, this is a logical way of refining the locus. As an example of what can be done, Fig. 3-4 shows a correction which was made to improve the shape of the lift portion of the locus. Both stations 12 and 1 on the locus are higher due to the corrected groove, and the resulting locus is now improved.

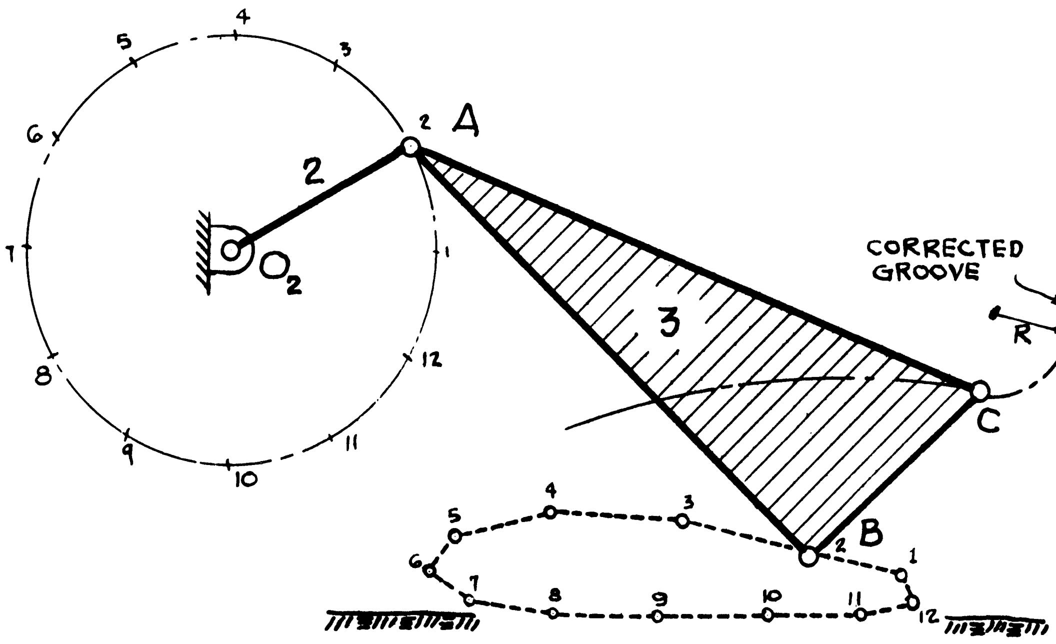


Fig. 3-4. A corrected cam-groove linkage.

Corrections to the velocity, and even to the acceleration, can also be made by correcting the cam groove. But, since velocity and acceleration are related to position, changing of one of these quantities will also affect the other two. Consequently, there is a limit to what can be done in this direction.

It is clear that the two groups of solutions described thus far leave much to be desired. The mechanism of Fig. 3-2 represents the general configuration of the linkage which has been sought, but the locus has a poor shape. The mechanism of Figs. 3-3 and 3-4 represent, probably, the best locus that can be obtained using a four-bar linkage; but the existence of the cam groove is a great disadvantage.

It is not at all obvious that the solution to this problem consists in taking a linkage similar to the one of Fig. 3-3, turning it upside down, and connecting another link to point B, having the foot on the opposite end. Such an arrangement makes possible the selection of a linkage from a much larger group in the Hrones-Nelson catalogue having much more satisfactory loci. One of the best of these, although there are many good ones, was synthesized, according to this idea, and the result is shown in Fig. 3-5. The basic linkage, as selected from the catalogue,

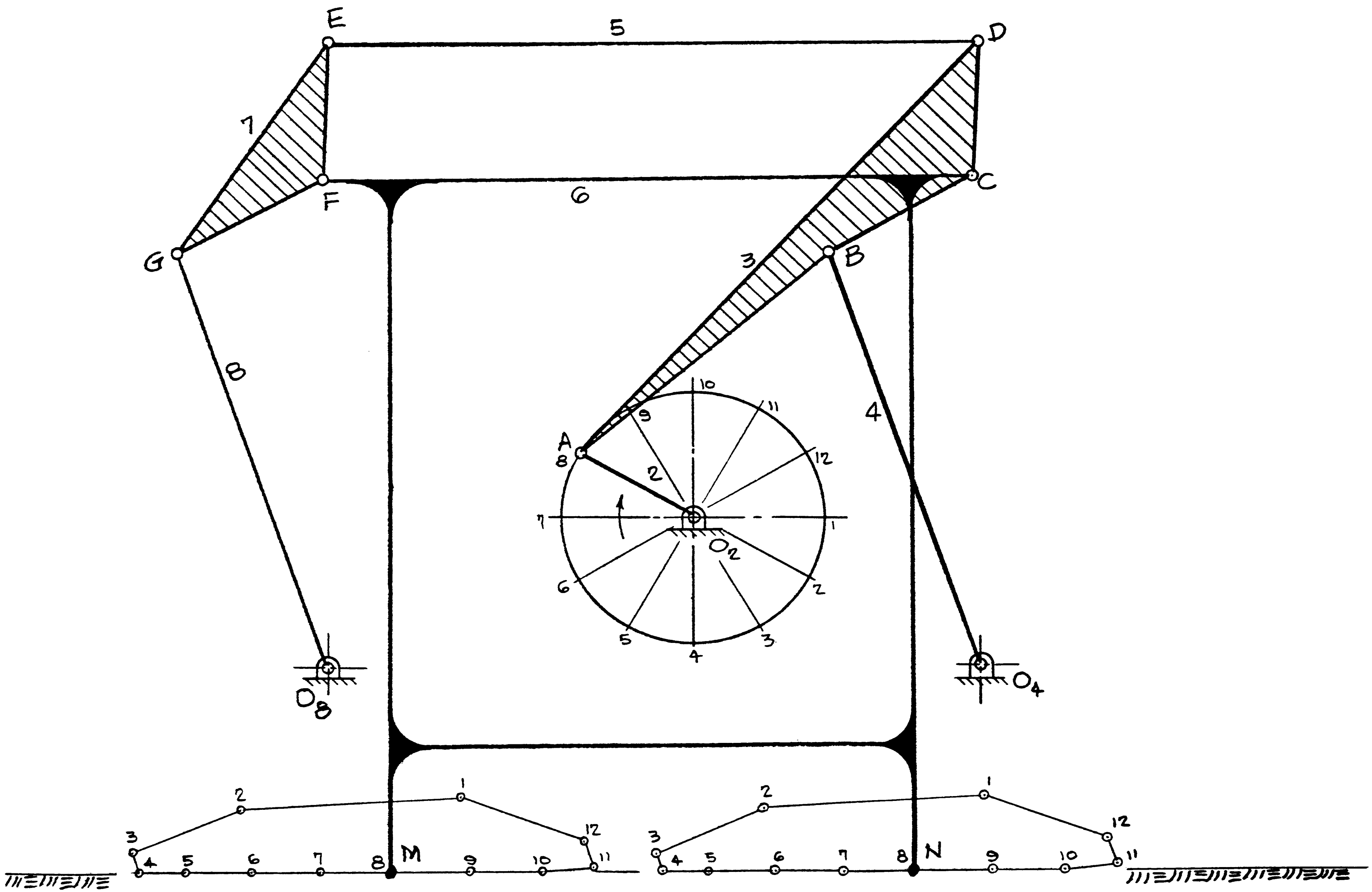


Fig. 3-5. The double-rocker mechanism.

consists of the chassis 1, the crank 2, the coupler 3 containing points A, B, and C, and the rocker 4. Point C on the coupler is the point to which we wish to connect the leg. But a leg connected only at C would not have constrained motion. Consequently, we borrow an idea from the transport mechanism, add links 5, 7, and 8 to the system, and get a linkage consisting of 2 feet M and N.

If we examine the loci traced out by the feet we find that the stride takes place during 210° of crank rotation, thus accomplishing the objective of getting a stride which occupies a major portion of the crank angle. In addition, the length of stride is 3-1/2 times the crank radius, which is very good. The spacing of the station numbers indicates that some velocity correction is necessary during stride, but this is no worse than any of the other linkages studied. The symmetry of the locus is about as good as one can expect to obtain with any linkage driven by a rotating crank. A higher return-stroke clearance can be obtained by slight alterations in the dimensions of the members, but this is obtained at the expense of the stride.

3-3. PERFORMANCE

In this section we shall discuss the performance of each of the three walking mechanisms in terms of how well they satisfy the requirements of Sec. 1-2. The numbers which follow identify the particular requirements of that section.

1. None of the three mechanisms described fulfill the uniform velocity requirement. They can all be made to do so by driving the cranks at non-uniform velocities using pairs of non-circular gears.

2. None of the strides are very long when compared to the overall space required by the mechanism. The linkage of Fig. 3-4 does the best job of satisfying this requirement. The linkage of Fig. 3-5 is not as bad as it appears because we are getting two feet instead of one with it.

3. In all three mechanisms the length of stride is fixed by the dimensions of the linkage. It cannot be controlled by the operator.

4. The height of step is satisfactory for the mechanisms of Figs. 3-4 and 3-5, but not for the one of Fig. 3-2.

5. In none of the mechanisms is the height of step controllable by the operator.

6. The stride-time to return-time ratios are as follows:

Fig. 3-2	Ratio	=	0.5
Fig. 3-4	Ratio	=	0.71
Fig. 3-5	Ratio	=	1.40

A ratio less than unity means that the stride occupies only a small portion of the cycle time. A ratio more than unity means that the stride occupies a major portion of the cycle time. Hence, Fig. 3-5 is superior.

7. If the cranks are driven from the same source of rotating power and a controlled differential is not employed, then the requirement of individual speed variation cannot be met. Using a controlled-differential arrangement, however, would make it possible to control the speeds of the feet on opposite sides of the vehicle for direction control.

8. All three mechanisms may be operated in either direction.

9. The inertia-force requirement is an important one requiring considerable analysis. The results depend upon the number of mechanisms and their arrangement on the vehicle. This requirement is treated in separate sections for the mechanisms of Figs. 3-4 and 3-5. The one of Fig. 3-2 is too poor to justify this lengthy analysis.

10. This requirement may be satisfied for each of the three mechanisms if a flywheel is present in the power source.

11. In none of the mechanisms is the height of the vehicle body above the terrain controllable by the operator.

In summary, the linkage of Fig. 3-2 can be made to fulfill four of the ten requirements discussed here, Fig. 3-4 will satisfy six of them, and Fig. 3-5 can be made to satisfy seven of them.

3-4. DYNAMIC ANALYSIS OF THE CAM-GROOVE MECHANISM

Our purpose in making a dynamic analysis is to obtain an idea of the inertia forces resulting from the action of a single mechanism, and then, by utilizing a number of mechanisms on the same vehicle, discover if these inertia forces can be balanced by proper phasing or timing of the several mechanisms relative to each other. Such an analysis will also reveal the energy variations which must be handled by the flywheel.

Since the methods of making a dynamic analysis are well known, only the results will be presented. The problem to be analyzed is stated as follows:

In Fig. 3-4, O_2A = 16 in., AC = 48 in., AB = 43.2 in., BC = 17.8 in.; radius of cam groove is 64 in. for stations 2 through 11 inclusive, and 6.4 in. for stations 12 and 1; the distance to the center of gravity of link 3 is 30 in. measured from A, and the weight is assumed to be 30 lb. Assuming this weight to be equally divided between points A, B, and C, the moment of inertia is I_3 = 3.28 lb-sec^2-ft. We shall assume that crank 2 is balanced and that it has an I_2 = 0.70 lb-sec^2-ft. Table 3-1 gives the angular velocities and accelerations of the crank for stations on the stride. This table was obtained by first finding a suitable set of dimensions for a pair of non-circular driving gears for the linkage. The figures given in the table correspond to a vehicle speed of 10 mph which was arbitrarily selected. The crank angular velocity for all other stations is 15.4 rad/sec, and the angular acceleration is zero.

After the design of an actual vehicle has been completed it may be found that some of these values are somewhat unrealistic. For the purposes of obtaining values to compare with other vehicles and walking mechanisms these assumptions are perfectly satisfactory. Furthermore our prime purpose is to discover if the inertia forces for a vehicle utilizing the walking mechanism of Fig. 3-4 can be balanced, and this can be done with any group of compatible data.

TABLE 3-1. CRANK VELOCITY AND ACCELERATION

Station No.	Angular Velocity, rad/sec	Angular Acceleration, rad/sec^2
7	15.4	0
8	12.3	-78
9	10.9	-17
10	11.1	60
11	15.4	0

The results of this analysis, which is quite lengthy, are tabulated in Table 3-2. The analysis was carried out for twelve time-equal steps. Results are given for one leg, two legs, and four legs, all mounted at the same place on the vehicle. In this example, the vehicle must have sixteen legs (minimum), four at each corner, in order that at least four feet be on the ground at all times.

TABLE 3-2. RESULTS OF DYNAMIC ANALYSIS OF MECHANISM OF FIG. 3-4

T = inertia torque, ft-lb
F^x = horizontal component of inertia force, lb
F^y = vertical component of inertia force, lb

Step No.	One Leg			Two Legs			Four Legs		
	T	F^x	F^y	T	F^x	F^y	T	F^x	F^y
1	-180	460	190	-225	300	135	-223	80	250
2	-175	250	70	-178	200	10	53	45	- 5
3	- 40	60	150	- 65	15	110	- 14	- 90	- 25
4	158	-150	150	22	-220	115	-223	80	250
5	141	-285	60	231	-155	- 15	53	45	- 5
6	- 32	-275	- 5	51	-105	-135	- 14	- 90	- 25
7	- 45	-160	- 55	-225	300	135	-223	80	250
8	- 3	- 50	- 60	-178	200	10	53	45	- 5
9	- 25	- 45	- 40	- 65	15	110	- 14	- 90	- 25
10	-136	- 70	- 35	22	-220	115	-223	80	250
11	90	130	- 75	231	-155	- 15	53	45	- 5
12	82	170	-130	51	-105	-135	- 14	- 90	- 25

The results for two legs are obtained from those for one leg by phasing the two 180° apart. Those for four legs were obtained by phasing the legs 90° apart. Data for six legs, equally phased, are not shown, but this can easily be calculated directly from the table.

It is probable that the peak values of these forces occur somewhere in between the steps chosen. Only enough steps were taken to obtain a general idea of the trend of the curves.

The table shows that the inertia-torque peaks increases as the number of legs increases. Of course, the integral of the product of the torque and the differential time is always zero over a complete cycle. The high peaks mean that a large flywheel is necessary to store the energy.

The component F^x is a force which shakes the vehicle in the forward and backward directions. This inertia force is reduced a great deal, for this mechanism, by employing four legs, and it would be reduced more than this if six legs were used at each corner.

The component F^y is the force that shakes the vehicle in the vertical direction. The table shows that this force increases as more legs are used.

We can conclude, from this analysis, that the inertia forces cannot be balanced for the mechanism of Fig. 3-4 and, hence, requirement 9 is not satisfied.

3-5. DYNAMIC ANALYSIS OF THE DOUBLE-ROCKER MECHANISM (FIG. 3-5)

A complete dynamic analysis of the linkage of Fig. 3-5 would involve the following:

1. Determining the angular velocities and accelerations for all eight links for 12 crank positions.

2. Calculations of the inertia forces and torques due to each link and their reactions on each of the other 7 links. This requires the superposition of 8 calculations for each crank position making a total of 96 lengthy calculations.

The calculations indicated in the previous paragraph are not difficult but they are lengthy and complicated, requiring graphical treatment and the making of many hundreds of free-body diagrams of the links in their various positions. Certainly, before proceeding with such a time-consuming project, it is wise to use every resource available to obtain a preliminary estimate of the probable value of such an analysis.

The first step, in getting this preliminary estimate, was a digital computer solution of the kinematics of links 3 and 4 of Fig. 3-5 with an angular velocity of Link 2 of 1 rad/sec. The results of this analysis, in increments of 5° of crank angle, are given in Table 3-3. The size of the linkage investigated and the nomenclature are illustrated in Fig. 3-6. The angle θ is measured counterclockwise from the x axis. Thus the crank is shown in the $\theta = 0$ position. All angular velocities and accelerations are positive in the counterclockwise direction. The quantities V_C and A_C are the velocity and acceleration of point C, respectively, in units of feet and seconds. The angles θ_V and θ_A are the angles that the velocity and acceleration vectors, respectively, make with the x axis.

Examining V_C and θ_V, of this table, it is easy to see that stride begins about where $\theta = 71.5°$ where $V_C = 0.73$ fps and $\theta_V = 188.2°$ because the foot is now nearly on a horizontal course. Similarly, stride ends at about $\theta = 251.5°$. Now, if we take a vehicle velocity of 14.7 fps (10 mph) and make V_C equal to this during stride, then the crank angular velocity will have

TABLE 3-3. DIGITAL COMPUTER ANALYSIS

θ	ω_3	ω_4	α_3	α_4	V_C	θ_V	A_C	θ_A	θ	ω_3	ω_4	α_3	α_4	V_C	θ_V	A_C	θ_A
1.5°	-.88	-.56	.51	.86	2.93	348.4	4.39	202.0	181.5°	.39	.24	.17	-.11	1.04	179.2	.14	22.5
6.5	-.83	-.49	.65	.93	2.59	344.3	4.55	191.8	186.5	.40	.23	.16	-.11	1.03	178.9	.14	21.3
11.5	-.77	-.40	.73	.94	2.23	340.2	4.52	183.8	191.5	.42	.22	.15	-.11	1.02	178.6	.14	19.0
16.5	-.70	-.33	.76	.92	1.87	336.1	4.34	177.5	196.5	.43	.21	.13	-.11	1.00	178.4	.13	15.7
21.5	-.63	-.25	.76	.87	1.53	331.7	4.07	172.4	201.5	.44	.20	.12	-.12	.99	178.3	.13	11.6
26.5	-.57	-.17	.74	.80	1.21	326.5	3.75	168.3	206.5	.45	.19	.11	-.12	.98	178.1	.14	7.2
31.5	-.50	-.10	.70	.73	.92	319.8	3.40	165.1	211.5	.46	.17	.09	-.12	.97	178.0	.15	3.2
36.5	-.44	-.05	.66	.66	.68	308.9	3.06	162.7	216.5	.47	.16	.07	-.12	.96	178.0	.17	0.1
41.5	-.39	0	.62	.59	.48	293.9	2.72	161.0	221.5	.47	.15	.06	-.13	.94	178.0	.19	358.3
46.5	-.34	.06	.57	.52	.36	267.6	2.40	169.9	226.5	.48	.14	.04	-.13	.92	178.0	.22	357.7
51.5	-.29	.10	.53	.46	.36	236.1	2.10	159.3	231.5	.48	.13	.01	-.13	.90	178.0	.26	358.4
56.5	-.24	.14	.50	.40	.43	213.3	1.83	159.2	236.5	.48	.12	-.01	-.14	.88	177.9	.31	0
61.5	-.20	.17	.46	.34	.53	200.2	1.58	159.6	241.5	.48	.10	-.04	-.15	.85	177.8	.36	2.1
66.5	-.16	.20	.43	.30	.63	192.7	1.36	160.3	246.5	.47	.09	-.07	-.16	.81	177.6	.44	4.7
71.5	-.13	.22	.41	.25	.73	188.2	1.16	161.5	251.5	.47	.08	-.10	-.18	.77	177.1	.53	7.5
76.5	-.09	.24	.38	.21	.81	185.3	.98	162.9	256.5	.46	.06	-.14	-.19	.72	176.3	.64	10.3
81.5	-.06	.26	.36	.18	.89	183.5	.82	164.7	261.5	.44	.04	-.19	-.22	.66	174.9	.77	12.9
86.5	-.03	.27	.34	.14	.95	182.3	.67	166.7	266.5	.42	.02	-.24	-.24	.59	172.5	.94	15.3
91.5	0	.29	.33	.12	1.00	181.5	.55	169.0	271.5	.40	0	-.30	-.28	.51	168.4	1.14	17.4
96.5	.03	.29	.32	.09	1.04	181.0	.44	171.6	276.5	.37	-.03	-.38	-.32	.42	161.0	1.39	19.1
101.5	.06	.30	.30	.07	1.07	180.8	.34	174.5	281.5	.33	-.06	-.47	-.37	.33	146.1	1.69	20.4
106.5	.08	.31	.29	.05	1.10	180.7	.25	177.6	286.5	.29	-.09	-.58	-.43	.27	116.5	2.05	21.1
111.5	.11	.31	.29	.03	1.12	180.6	.18	181.3	291.5	.23	-.13	-.71	-.50	.32	78.3	2.48	21.3
116.5	.13	.31	.28	0	1.13	180.7	.11	186.0	296.5	.16	-.18	-.86	-.58	.49	54.2	2.98	20.8
121.5	.15	.31	.27	0	1.14	180.7	.06	194.0	301.5	.08	-.23	-1.03	-.68	.74	41.8	3.54	19.5
126.5	.18	.31	.26	0	1.14	180.8	0	236.6	306.5	-.02	-.30	-1.21	-.77	1.06	34.6	4.12	17.4
131.5	.20	.31	.26	-.04	1.14	180.8	.04	350.7	311.5	-.13	-.37	-1.39	-.86	1.43	29.7	4.67	14.1
136.5	.22	.30	.25	-.05	1.14	180.8	.07	2.6	316.5	-.26	-.45	-1.53	-.91	1.84	25.6	5.10	9.5
141.5	.24	.30	.24	-.06	1.13	180.8	.09	7.9	321.5	-.40	-.53	-1.60	-.91	2.27	21.8	5.27	3.1
146.5	.26	.29	.24	-.07	1.12	180.7	.11	11.7	326.5	-.54	-.60	-1.57	-.82	2.70	18.1	5.10	354.2
151.5	.29	.29	.23	-.08	1.11	180.6	.13	14.8	331.5	-.67	-.67	-1.41	-.64	3.07	14.1	4.57	341.3
156.5	.30	.28	.22	-.08	1.10	180.4	.14	17.3	336.5	-.78	-.71	-1.13	-.38	3.36	10.0	3.84	322.3
161.5	.32	.27	.21	-.09	1.09	180.2	.15	19.5	341.5	-.86	-.73	-.76	-.06	3.51	5.7	3.24	294.5
166.5	.34	.26	.20	-.10	1.07	180.0	.15	21.1	346.5	-.91	-.72	-.37	.24	3.54	1.3	3.17	262.0
171.5	.36	.25	.19	-.10	1.06	179.7	.15	22.3	351.5	-.93	-.69	-.01	.52	3.43	356.9	3.55	234.7
176.5	.38	.24	.18	-.10	1.05	179.5	.15	22.8	356.5	-.91	-.63	.29	.73	3.22	352.5	4.04	215.5

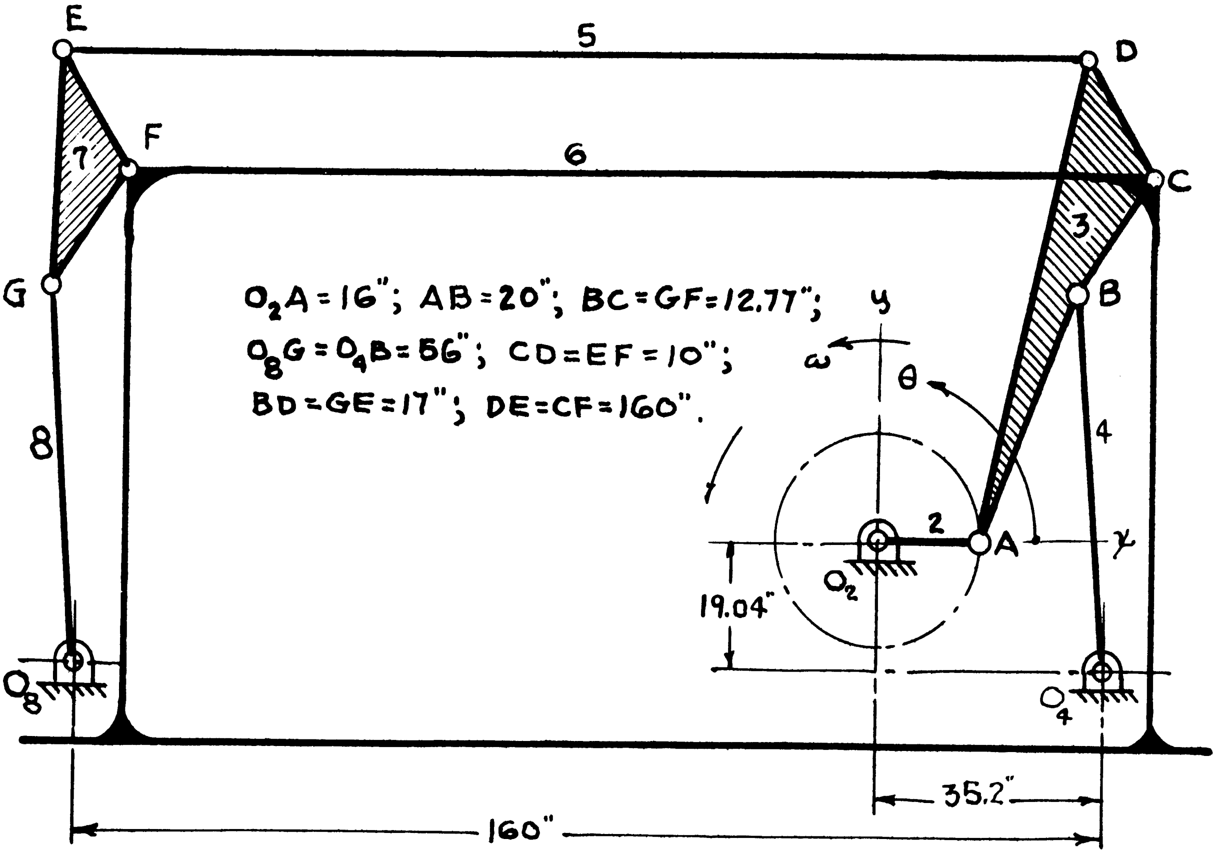


Fig. 3-6. Dimensions of the double-rocker mechanism.

to be changed to accomplish this. The formula is

$$\omega_2 = \frac{14.7}{V_C} \tag{3-1}$$

where ω_2 is the crank velocity in rad/sec. Listing crank angles from 71.5° to 251.5° in 10° intervals in Table 3-4, we may calculate the corresponding values of ω_2 in order to give a vehicle velocity of 14.7 fps. These values are listed in the second column.

The next step is to synthesize a pair of non-circular gears which will give the angular velocities shown in Table 3-4. Designating the driving gear by the subscript 3 and the driven gear by 2, the following relationship must hold:

$$r_2\omega_2 = r_3\omega_3 \tag{3-2}$$

$$r_2 + r_3 = K \tag{3-3}$$

$$C_2 = C_3 \tag{3-4}$$

TABLE 3-4. DATA FOR NON-CIRCULAR GEARS

θ	r_2	r_3	ω_2	θ	r_2	r_3	ω_2
1.5	1.78	2.22	21.2	181.5	2.18	1.82	14.2
11.5	1.72	2.28	22.6	191.5	2.16	1.84	14.4
21.5	1.68	2.32	23.4	201.5	2.13	1.87	14.9
31.5	1.65	2.35	24.2	211.5	2.11	1.89	15.2
41.5	1.64	2.36	24.4	221.5	2.09	1.91	15.6
51.5	1.67	2.33	23.7	231.5	1.98	1.96	16.3
61.5	1.73	2.27	22.3	241.5	1.88	2.02	17.3
71.5	1.83	2.17	20.2	251.5	1.88	2.12	19.1
81.5	2.03	1.97	16.5	261.5	1.88	2.12	19.2
91.5	2.14	1.86	14.7	271.5	1.88	2.12	19.2
101.5	2.21	1.79	13.7	281.5	1.88	2.12	19.2
111.5	2.27	1.73	13.0	291.5	1.88	2.12	19.2
121.5	2.27	1.73	12.9	301.5	1.88	2.12	19.2
131.5	2.27	1.73	12.9	311.5	1.88	2.12	19.2
141.5	2.27	1.73	13.0	321.5	1.88	2.12	19.2
151.5	2.25	1.75	13.2	331.5	1.88	2.12	19.2
161.5	2.23	1.77	13.5	341.5	1.88	2.12	19.2
171.5	2.20	1.80	13.9	351.5	1.83	2.17	20.2

In these equations r is the instantaneous pitch radius, ω the angular velocity, K a constant, and C the circumference. The first equation states that the pitch-line velocities are equal. Equation (3-3) states that the center distance must remain constant. And Eq. (3-4) states that the pitch curves must have the same circumferences. Solving Eqs. (3-2) and (3-3) simultaneously yields

$$r_2 = \frac{K}{\frac{\omega_2}{\omega_3} + 1} \qquad (3\text{-}5)$$

The procedure for solving Eq. (3-5) is to decide on a suitable center distance K, and then to try various values of ω_3 (a constant) until Eq. (3-4) is satisfied. This is a graphical solution.

Figure 3-7 shows how the method works. Choose a center distance K = O_2O_3. Guess at ω_3. Calculate r_2 equal to O_2a, O_2b, O_2c, etc., corresponding to the desired values of ω_2 and lay these off as shown using angular intervals, in this case, of 10°. The radius r_3 is the difference between K and r_2, so these values may be set on a compass and the arcs O_3b', O_3c', O_3d', etc., laid off. Now, find points on the pitch curve by constructing the

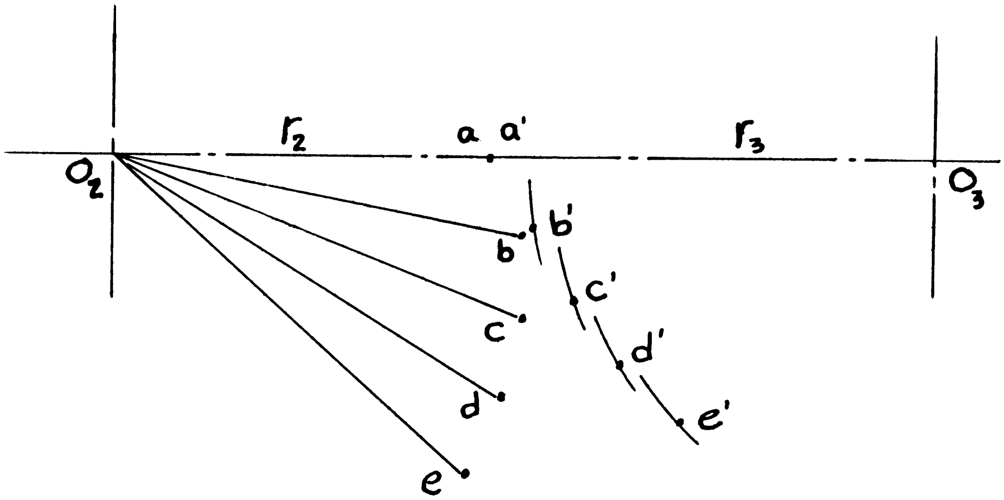


Fig. 3-7. Synthesis of non-circular gears.

equalities ab = a'b', bc = b'c', cd = c'd', etc. Continue this procedure until the entire gear has been traversed. Usually it will be found that the guess for ω_3 is wrong and the pitch curve for gear 3 does not close, or that it overlaps itself. If it does not close, try a larger value for ω_3. If it overlaps, try a smaller value.

Using this procedure, the pair of gears shown in Fig. 3-8 were synthesized. Since stride occurs between 71.5° and 251.5°, these radii are fixed. However, continuing from 251.5°, back around to 71.5°, we have a curve which can be faired in to suit. The results of this synthesis are given in Table 3-4 together with the values of ω_2 that result.*

The next step in our analysis is to find the incremental times required for the crank to traverse each 10° increment. We do this by averaging the angular velocities at the beginning and end of each interval, and dividing this average velocity into 0.1745 radians, the radian measure

*The Land Locomotion Laboratory of the Detroit Arsenal constructed a pair of non-circular gears from data obtained by a rough graphical analysis. The computer solution described earlier in this section makes possible a much more satisfactory solution to this problem. It is believed that the pair of gears synthesized here will overcome many of the deficiencies which were undoubtedly found in the earlier ones.

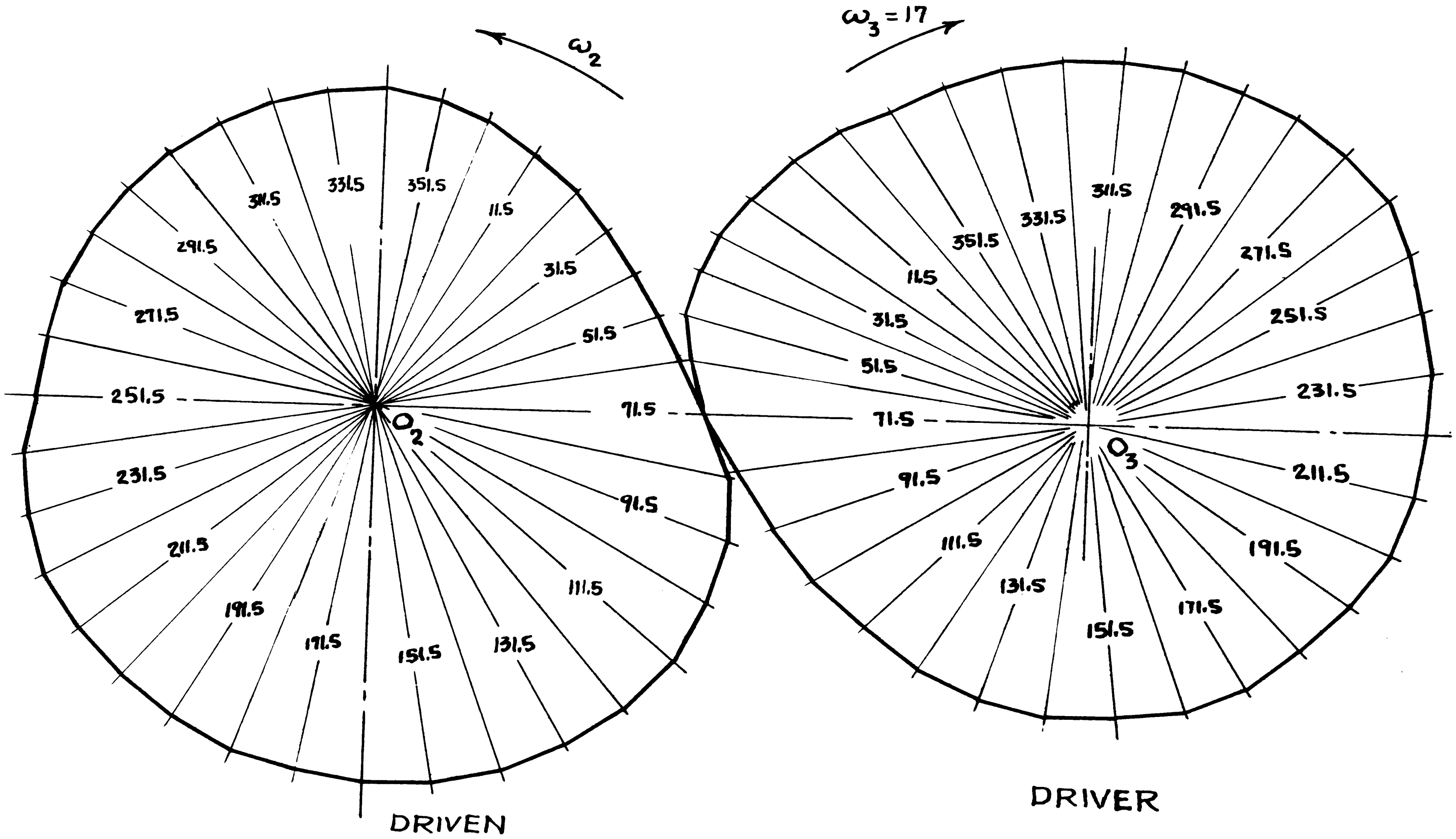


Fig. 3-8. Non-circular gears.

of 10°. Thus the column headed Δt in Table 3-5 is the result of these calculations. Summing these gives the total time t, in the next column. Then the values of A_C and θ_A are merely copied from Table 3-3. The quantities $A_C \cos \theta_A$ and $A_C \sin \theta_A$ are the components of the acceleration in the x and y directions, respectively.

From Table 3-5 we see that the time of one cycle (period) is about 0.368 sec. The time for stride is found to be about 0.216 sec. Therefore, stride occupies 59 per cent of the cycle.

We have pointed out, earlier, the difficulty of making a complete dynamic analysis of the double-rocker mechanism. In this particular case, a great deal of weight will be present in the legs, and so we can get a good idea of whether or not the linkages can be balanced simply by trying to balance the accelerations against one another. The peak acceleration, in this example, is 2,310 fps^2. If this operates on a leg weighing 60 lb, then the resulting inertia force is

$$F = -mA = -\frac{60}{32.2}(2,310) = -4,300 \text{ lb}$$

This is probably equal to, or even more than, the weight of a vehicle that could be supported by a 60 lb leg.

It should be noted that the accelerations in Table 3-5 are in slight error because of the fact that the crank does have angular acceleration, and these accelerations were computed assuming a constant angular velocity for each phase of the linkage. Since the foot is moving at a constant velocity, it really has no acceleration during stride. Yet Table 3-5 shows a small acceleration present. If the angular acceleration of the crank were considered, then A_C during stride would be zero. Since the crank changes its angular velocity slowly, we are probably safe in making this assumption.

Having calculated quantities ($A_C \cos \theta_A$ and $A_C \sin \theta_A$) which are proportional to the horizontal and vertical components of the inertia forces, let us plot these on a time scale and see what can be done about balancing such a linkage. Figure 3-9 is such a plot. The superscripts x and y designate the acceleration components in the x and y directions, respectively. We attempt to achieve balance by visualizing such curves superimposed on one another and phased so that the negative accelerations added to the positive ones give zero at all times. At the same time, in phasing these, we must be sure that one foot is on the ground at all times.

TABLE 3-5. ACCELERATIONS OF THE DOUBLE-ROCKER MECHANISM

θ	ω_2	ω_{av}	Δt	t	A_C	θ_A	$A_C \cos \theta_A$	$A_C \sin \theta_A$
1.5	21.2			0	1,970	202.0	-1,825	- 740
11.5	22.6	21.9	0.00797	0.00797	2,310	183.8	-2,300	- 155
21.5	23.4	23.0	0.00759	0.01556	2,230	172.4	-2,210	290
31.5	24.2	23.8	0.00733	0.02289	2,190	165.1	-2,120	565
41.5	24.4	24.3	0.00718	0.03007	1,620	161.0	-1,530	530
51.5	23.7	24.1	0.00725	0.03732	1,180	159.3	-1,100	420
61.5	22.3	23.0	0.00759	0.04491	780	159.6	- 730	270
71.5	20.2	21.2	0.00823	0.05314	470	161.5	- 445	150
81.5	16.5	18.4	0.00948	0.06262	220	164.7	- 210	60
91.5	14.7	15.6	0.01120	0.07382	120	169.0	- 118	23
101.5	13.7	14.2	0.01230	0.08612	60	174.5	- 60	5
111.5	13.0	13.4	0.01303	0.09915	30	181.3	- 30	0
121.5	12.9	12.9	0.01352	0.11267	10	194.0	- 10	2
131.5	12.9	12.9	0.01352	0.12619	0	350.7	0	0
141.5	13.0	13.0	0.01342	0.13961	15	7.9	15	2
151.5	13.2	13.1	0.01333	0.15294	20	14.8	20	5
161.5	13.5	13.4	0.01303	0.16597	25	19.5	24	8
171.5	13.9	13.7	0.01272	0.17869	30	22.3	28	11
181.5	14.2	14.1	0.01238	0.19107	30	22.5	28	12
191.5	14.4	14.3	0.01220	0.20327	30	19.0	28	10
201.5	14.9	14.7	0.01187	0.21514	30	11.6	29	6
211.5	15.2	15.1	0.01154	0.22668	35	3.2	35	0
221.5	15.6	15.2	0.01148	0.23816	45	358.3	45	0
231.5	16.3	15.9	0.01098	0.24914	70	358.4	70	0
241.5	17.3	16.8	0.01039	0.25953	110	2.1	110	0
251.5	19.1	18.7	0.00934	0.26887	190	7.5	190	25
261.5	19.2	19.1	0.00913	0.27800	280	12.9	270	60
271.5	19.2	19.2	0.00908	0.28708	420	17.4	400	125
281.5	19.2	19.2	0.00908	0.29616	620	20.4	580	215
291.5	19.2	19.2	0.00908	0.30524	920	21.3	855	335
301.5	19.2	19.2	0.00908	0.31432	1,400	19.5	1,320	470
311.5	19.2	19.2	0.00908	0.32340	1,720	14.1	1,700	420
321.5	19.2	19.2	0.00908	0.33248	1,940	3.1	1,930	105
331.5	19.2	19.2	0.00908	0.34156	1,680	341.3	1,590	- 540
341.5	19.2	19.2	0.00908	0.35064	1,200	294.5	500	-1,090
351.5	20.2	19.7	0.00886	0.35950	1,440	234.7	- 830	-1,170
361.5	21.2	20.7	0.00843	0.36793	1,970	202.0	-1,825	- 740

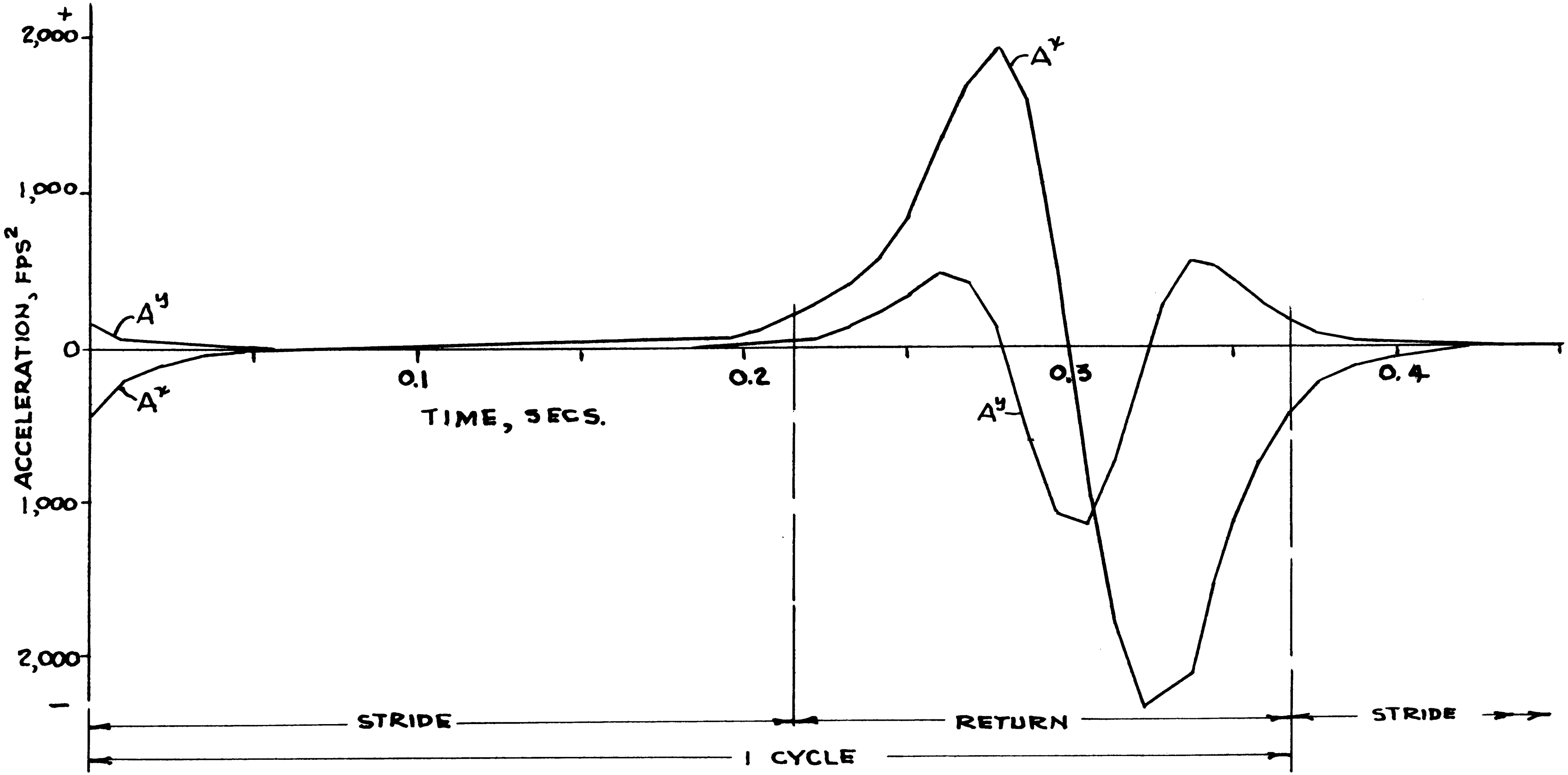


Fig. 3-9. Acceleration-time diagram.

For example, suppose a two-legged vehicle. Imagine that Fig. 3-9 is reproduced on a sheet of tracing paper, and that we are making the time axes coincident and sliding the traced curve along the time axis of Fig. 3-9. We are hunting a place where the negative A^x's and A^y's are opposite the positive ones. We must also be sure that stride exists continuously. The last requirement means that the return stroke of one leg will have to occur during stride of the other leg. Consequently we can obtain no balancing at all using two legs.

If we decide to employ four legs, in gangs of two, all mounted on the same side of the vehicle, but with one gang at the front and the other gang at the rear, then we can partially balance the x components. If the rear gang is phased so that the negative peaks coincide with the positive peaks of the front gang, then some degree of balance is obtained. It is not complete though, because of the shape of the positive and negative curves. The y components, with this scheme, will not be balanced because the lines of action are not coincident.

There is no way of phasing legs on opposite sides of the vehicle to obtain balance of the inertia forces because neither of the components have coincident lines of action. It is possible, as we shall see later, to phase the legs on opposite sides of the vehicle to balance the inertia torques. The lack of a complete analysis, in this example, has prevented the calculation of these torques.

It is clear from this simplified analysis that requirement 9 cannot be satisfied. It is unnecessary, therefore, to make the complete dynamic analysis.

3-6. VEHICLE GEOMETRY

The phasing of several walking linkages to reduce or eliminate the shaking and rocking of the vehicle due to the inertia forces and inertia torques has already been discussed. We are now interested in the arrangement of the mechanisms on the vehicle itself.

Figure 3-10 is a side view of a vehicle having legs (not shown) mounted at points A and B. If the legs are properly phased, then, at some instant in time, the inertia torque T_A will be counterclockwise and the torque T_B will be clockwise. Since a torque vector is a free vector, these torques will cancel one another, if their magnitudes are equal, regardless of the location of points A and B.

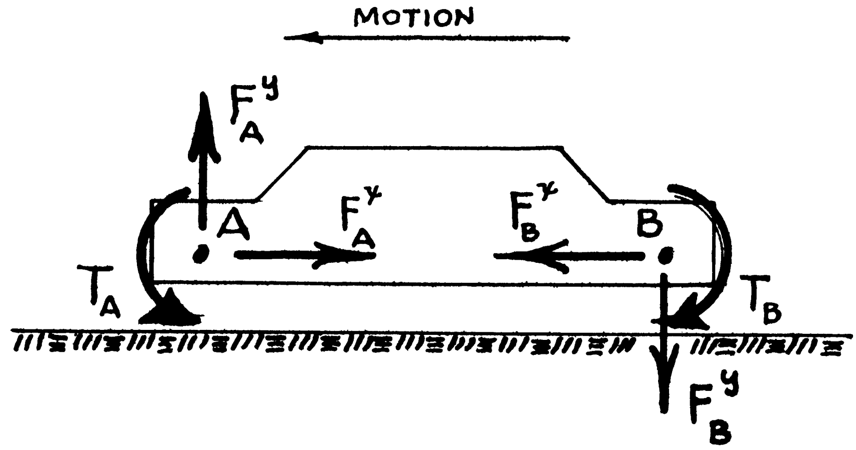


Fig. 3-10. Forces on the vehicle.

Also shown in the figure are the horizontal components F_A^x and F_B^x of the inertia forces due to the action. Again we see that these forces will cancel each other, if their magnitudes are equal, and if their elevation on the vehicle are the same. Other than elevation, the location of A and B on the vehicle is not important in order to achieve cancellation.

Finally, examining the vertical components F_A^y and F_B^y, we see that these forces form a couple which will tend to rock or oscillate the vehicle about a horizontal axis through the side of the vehicle. In order to achieve cancellation of the vertical components it is necessary for points A and B to be coincident.

Thus it is necessary to mount enough properly phased legs <u>at one point</u> to balance the vertical components of the inertia forces; and it is necessary to mount enough properly phased legs <u>on one side</u> of the vehicle in order to balance the inertia torques and the horizontal components of the inertia forces. Aside from considerations of support and traction these are the guiding principles which must be used in selecting and arranging the legs. A machine employing perfect walking mechanisms would have a minimum of eight legs on each side, four at A and four at B, properly phased, in order that the resultant inertia forces and torques be zero at all times.

3-7. CONTROL

The stride length and height of step are fixed by the linkage dimensions for purely mechanical mechanisms, and consequently these cannot be controlled.

The speed of walking machines can be controlled in the usual manner as for wheeled and tracked vehicles.

In order to obtain control of direction, for a single vehicle, it is necessary to employ differentials connecting the driving mechanisms of the legs on opposite sides of the vehicle, and then slow the action on one side relative to the other. Only the usual mechanical design problems would be encountered in providing this means.

4. HYDRAULICALLY OPERATED WALKING MECHANISMS

4-1. DESCRIPTION

Hydraulic operation was first considered as a means of overcoming the deficiencies of the mechanisms which were driven by a rotating crank. We have already seen that these deficiencies include the lack of control over the height of step, and an imperfect locus resulting in unbalanced inertia forces. Accordingly, the mechanism of Fig. 4-1 uses a hydraulic cylinder for proper positioning of the foot during all phases of the action. Point C is the foot, link 2 the crank, and link 5 the cylinder.

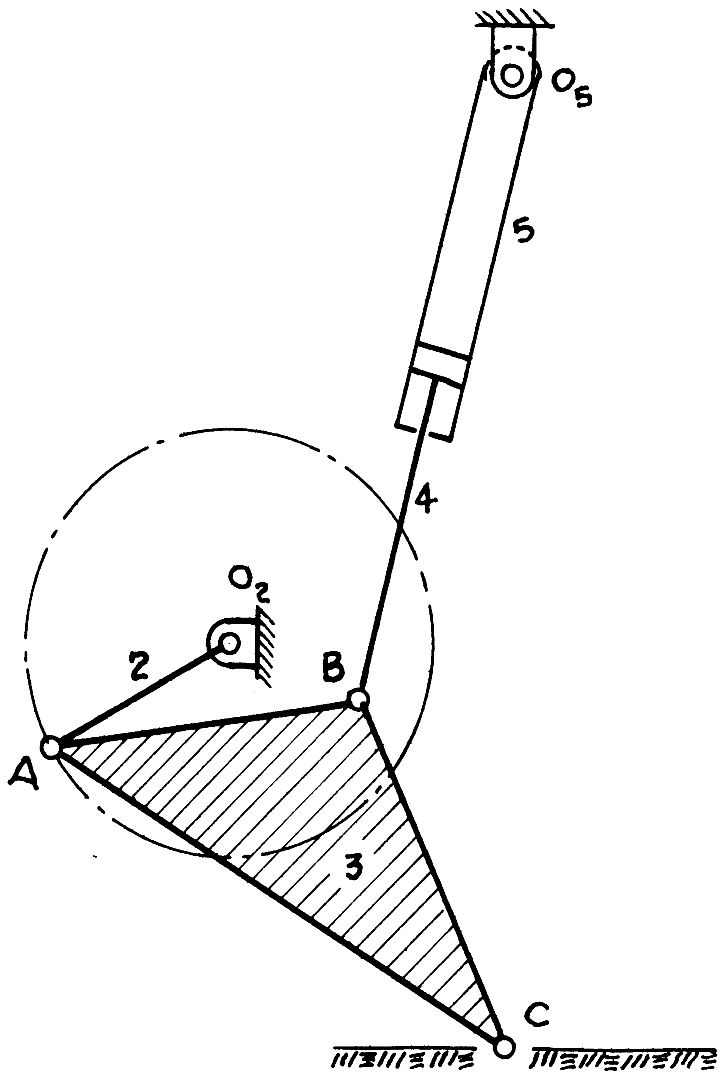


Fig. 4-1. A hydraulic walking linkage.

The cylinder is pivoted at O_5, and the piston is connected to link 4. Thus, when the piston is stationary relative to the cylinder, links 4 and 5 constitute an equivalent single link which oscillates about O_5. This then becomes the same class of linkages as the one of Fig. 3-2.

The addition of the hydraulic cylinder does give control over the height of step. But the locus generated is very poor and would result in very high acceleration forces on the vehicle because of the difficulty of obtaining balance by phasing several mechanisms. Furthermore, the crank must still rotate at a non-uniform angular velocity. Consequently, we have not gained anything by taking this step.

If the hydraulic power is going to be available on the vehicle, one might as well go "whole-hog," and use it both for driving and lifting the feet. Using one cylinder for driving and another cylinder, attached to the same mechanism, for lifting, then it ought to be possible to program the action of the two cylinders to obtain any desired locus. The simplest possible arrangement of two cylinders to accomplish this purpose is the one shown in Fig. 4-2. Here cylinder 2 is intended to take care of the driving function, and cylinder 5 the lifting function. By programming or controlling the flow to these two cylinders properly, the foot B can be made to generate the ideal locus. Unfortunately, the driving cylinder and piston rod are placed in bending by the weight of the vehicle, and so this is not a good solution.

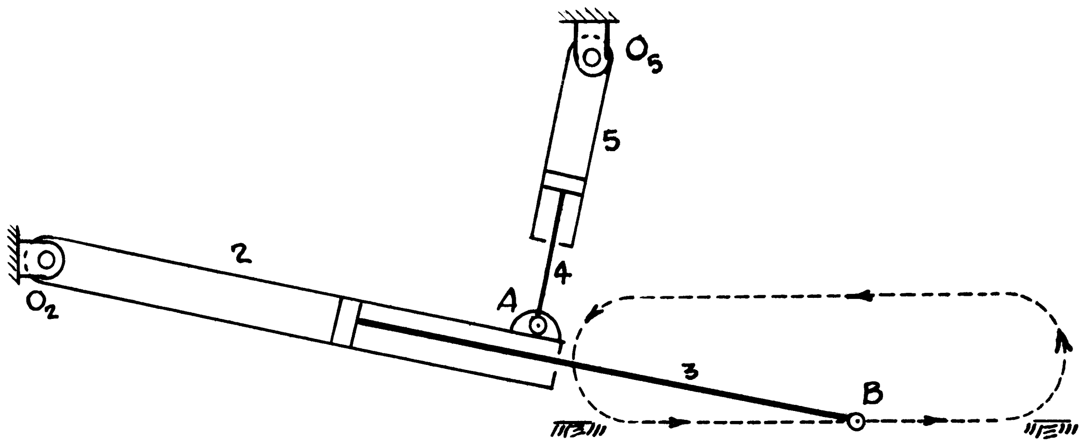


Fig. 4-2. A two-cylinder walking linkage.

In an attempt to eliminate the bending on the piston rods, the mechanism of Fig. 4-3 was devised. Actually, this is one of the best of a large class of mechanisms in which rigid links are utilized in combination with two cylinders. In this figure cylinder 2 is intended to accomplish the driving function, and cylinder 5 the lifting function. The mechanism

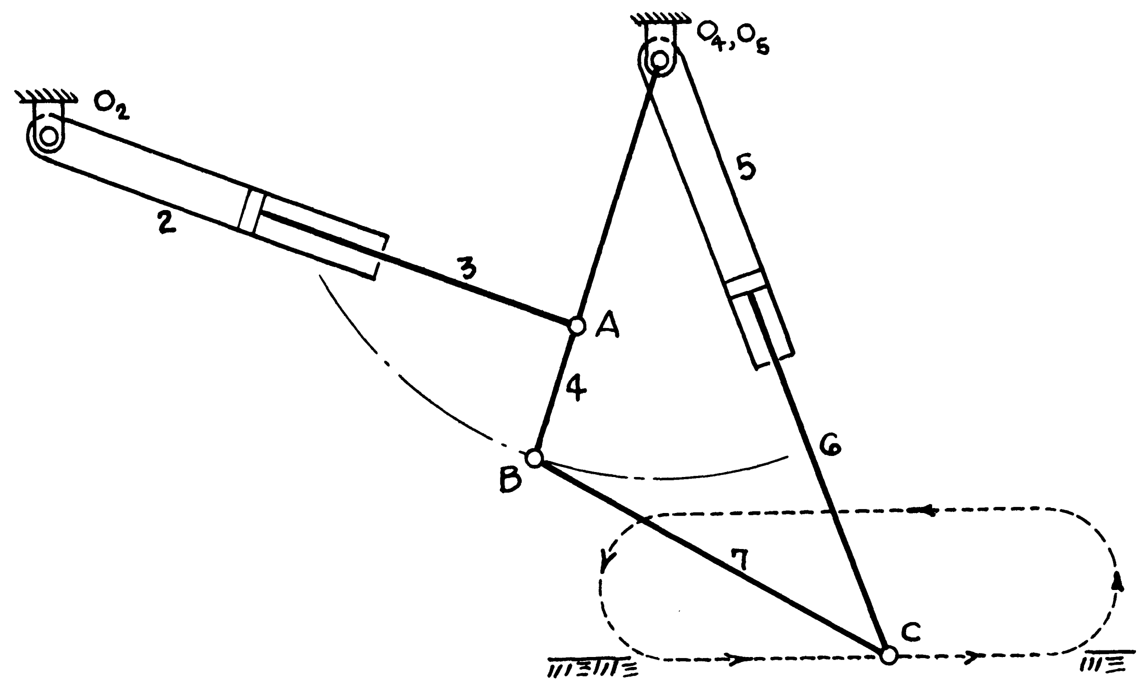


Fig. 4-3. Improved model.

is quite versatile. For example, the one of Fig. 4-3 has the piston rod of the driving cylinder connected to rocker 4 between point B and the pivot point O_4. Thus, the piston motion is less than the length of stride. So, in effect, we are multiplying the piston motion by a number greater than unity to get the stride length. But, if the piston rod is connected to the rocker outside of point B, then we get the reverse, and the piston motion is now less than the stride.

In addition there is really no need to connect the piston rod of the lifting cylinder at point C. It can be connected to link 7 anywhere between B and C. Thus we can obtain multiplying action for the lifting cylinder too.

We may note that points O_4 and O_5 of Fig. 4-3 need not be coincident. For example, if O_5 is located on the vertical center line of the locus, then the rocking motion of the lift cylinder will be symmetrical about this center line.

One of the difficulties of these two-cylinder mechanisms (Figs. 4-2 and 4-3) is that the piston motions for lifting and driving are not independent of each other. For example, if the height of step is made one-fourth of the stride, then a certain set of programs must be used to control the action of the two cylinders. But if the height of step is changed, then a different program for each cylinder must be used. Because of this dependence it may well be impossible to cause the cylinders to faithfully reproduce their respective programs. If they do not, the locus becomes misshapen, and the inertia forces on the vehicle become unbalanced.

Although the use of a hydraulic mechanism has many advantages over the mechanical mechanisms investigated, the dependence of the two cylinders creates a somewhat unsatisfactory solution. Consequently we look for a two-cylinder mechanism in which the lift and drive functions are completely independent.

In the pantograph mechanism (Fig. 4-4) the piston of the drive cylinder drives the slider 9 backwards and forwards on a line parallel to the vehicle motion. The lift cylinder drives the slider at the pantograph center O in a completely vertical up-and-down motion. The foot is at A. Links 3 and 4 extend from B to E and from C to D, respectively. Varying the stroke of the lift cylinder varies only the height of step. Varying the stroke of the drive cylinder changes only the length of stride. The motions are therefore completely independent. The cylinders may have any motion when coming to the ends of their strokes, parabolic, simple harmonic, or polynomial. So long as this motion is symmetrical at the two ends of the stroke, the locus is ideal because it will be symmetrical about both the horizontal and vertical center lines. This means that the valves can give the pistons any kind of accelerating and decelerating motion at the ends of the stroke; if these two motions are the same, the locus is symmetrical.

Another advantage of the pantograph mechanism is that it can be designed for any desired ratio between the piston strokes and the stride length and step height. The one of Fig. 4-4 has been designed so that the stroke of the drive piston is 75 per cent of the stride length.

4-2. DYNAMIC ANALYSIS OF THE PANTOGRAPH MECHANISM

We shall first present a static force analysis considering only the weight of the vehicle. In Fig. 4-5 let P be the force of the roadway against the foot. Due to this force, vertical reactions must be exerted by the frame of the vehicle against the linkage at points O and E. Designate these forces as R and F, respectively. Measuring the coordinate x positively to the right from the vertical center line, and taking moments about O gives

$$\Sigma M_O = Px - 0.75\, x F = 0$$

or

$$F = 1.333\, P$$

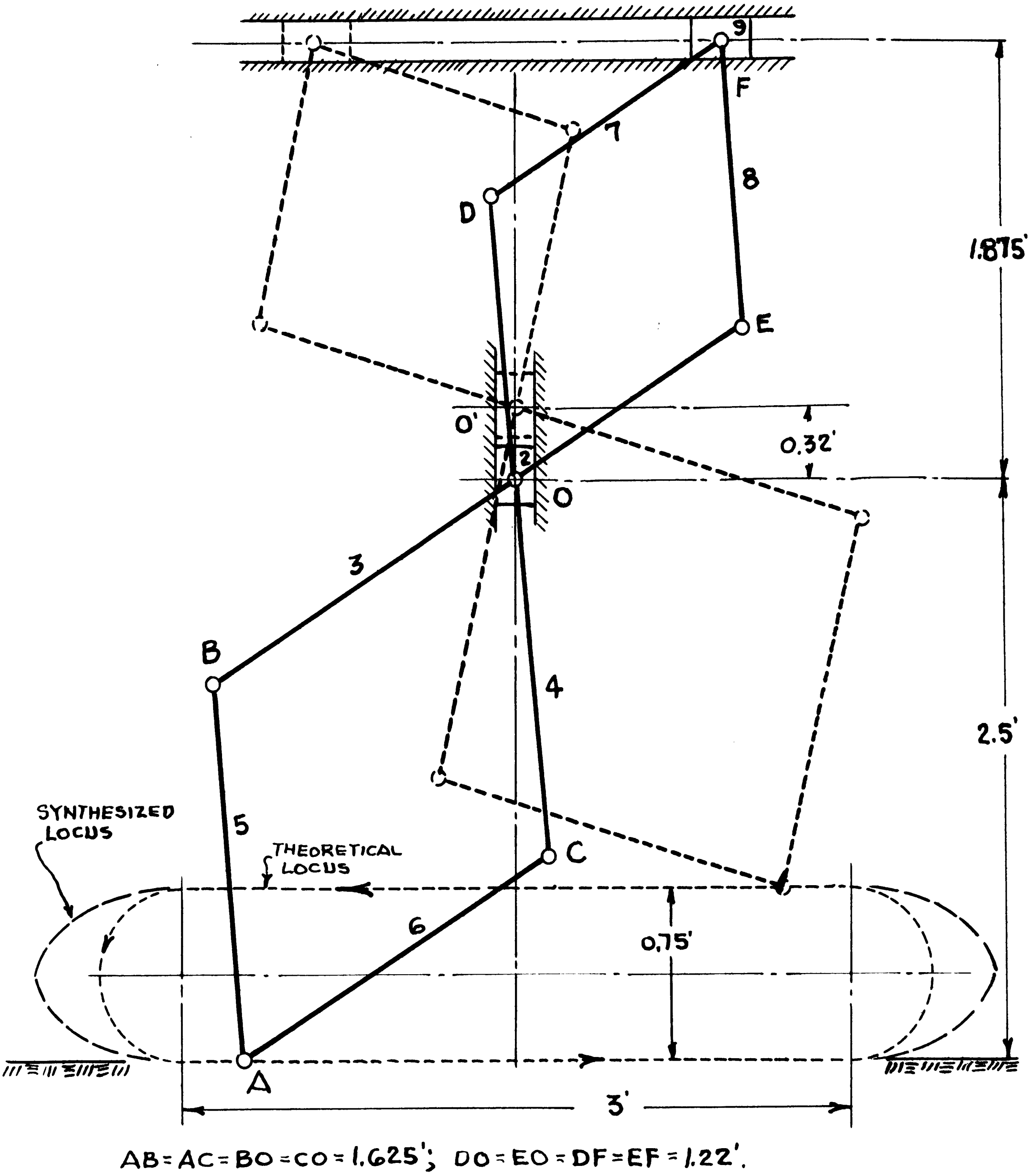


Fig. 4-4. The pantograph mechanism.

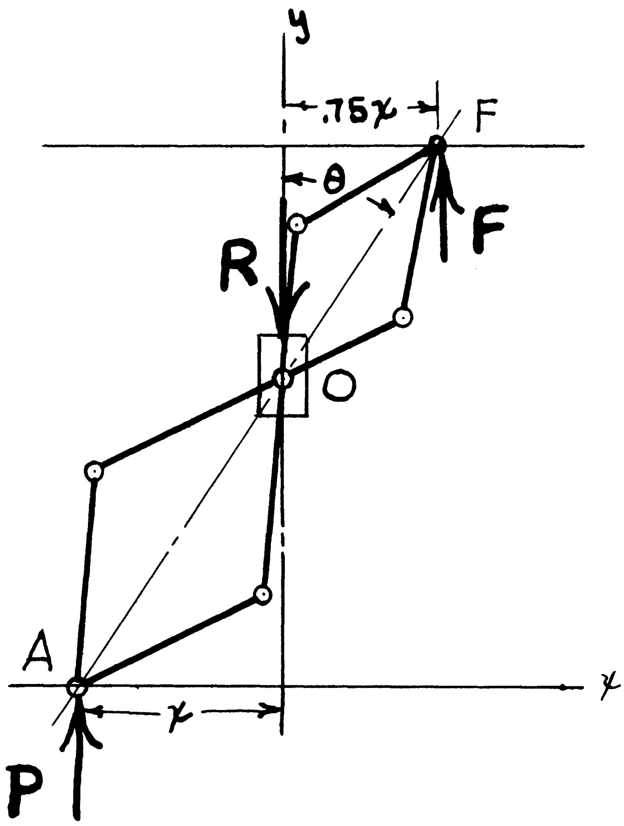


Fig. 4-5. Forces on the pantograph mechanism.

These are for the linkage dimensions of Fig. 4-4. Note that these forces are constant, and do not depend on the angle θ. Of course, the forces of the linkage on the frame are the negative of these. That is, the linkage pushes up against the frame at O, and pulls down on the frame at point F.

We begin the dynamic analysis by stating the problem to be solved. Using the dimensions of Fig. 4-4 we choose to analyze for a stride of 3 ft and a lift of 0.75 ft with a vehicle velocity of 29.4 fps (20 mph). The four phases of the action, stride, lift, return, and lower, are to be of equal duration. Thus, if four legs are mounted at one corner of the vehicle and phased one-quarter period apart, we hope that all inertia forces and torques will cancel one another. It is possible to balance the linkage so that the mass center is at O, and we shall assume this. With this assumption the inertia forces in the horizontal direction are always zero (except, of course, when the vehicle accelerates) because point O can move only in the vertical direction. We choose a weight of 30 lb for the leg.

Though the leg is balanced, it does have inertia, and the moment of inertia varies, depending on the position of the foot. The following formula gives reasonable values for the moment of inertia:

$$I = 0.199 \frac{x^2}{\sin^2 \theta} \tag{4-1}$$

where the meaning of x and θ are shown on Fig. 4-5. During stride these quantities are related by the equation

$$\theta = \tan^{-1} \frac{x}{2.5} \tag{4-2}$$

Differentiating Eq. (4-2) gives the angular velocity of the equivalent mechanism (Sec. 2-5). Thus

$$\dot{\theta} = \frac{\dot{x}/2.5}{1 + x^2/6.25} \tag{a}$$

where $\dot{\theta}$ and $\dot{x}$ are the velocities in the Newton notation. During stride $\dot{x} = 29.4$ fps and so Eq. (a) becomes

$$\dot{\theta} = \frac{11.75}{1 + 0.16x^2} \tag{4-3}$$

Differentiating Eq. (a) gives the angular acceleration

$$\ddot{\theta} = \frac{- \frac{\dot{x}}{2.5}\left(2 \frac{x\dot{x}}{6.25}\right)}{\left(1 + \frac{x^2}{6.25}\right)^2} = -0.8x(\dot{\theta})^2 \tag{4-4}$$

We shall consider $\dot{\theta}$ and $\ddot{\theta}$ as positive when the direction is counterclockwise.

Using Eqs. (4-1) to (4-4) we can now solve for the force to be delivered by the drive cylinder, and the inertia torque. The results of this analysis are shown in Table 4-1. Referring to Fig. 4-6 and taking a summation of the torques about point O, we see that

$$F = \frac{I\ddot{\theta}}{1.875} \tag{4-5}$$

A positive sign for F in Table 4-1 means that the piston force is in the same direction as the piston motion.

The next step is the analysis and synthesis of the lift action. The time duration of a single stride is the distance divided by the velocity.

TABLE 4-1. DYNAMICS DURING STRIDE

x	θ, deg	$\dot{\theta}$	$\ddot{\theta}$	I	$I\ddot{\theta}$	F
-1.50	31.0	8.64	89.5	1.70	152	81
-1.25	25.6	9.40	88.2	1.67	147	78
-1.00	21.8	10.10	81.7	1.45	118	63
-0.75	16.7	10.80	70.0	1.36	95	51
-0.50	11.3	11.30	51.0	1.30	66	35
-0.25	5.7	11.73	27.4	1.25	34	18
0	0	11.75	0	1.24	0	0
0.25	5.7	11.73	-27.4	1.25	- 34	-18
0.50	11.3	11.30	-51.0	1.30	- 66	-35
0.75	16.7	10.80	-70.0	1.36	- 95	-51
1.00	21.8	10.10	-81.7	1.45	-118	-63
1.25	25.6	9.40	-88.2	1.67	-147	-78
1.50	31.0	8.64	-89.5	1.70	-152	-81

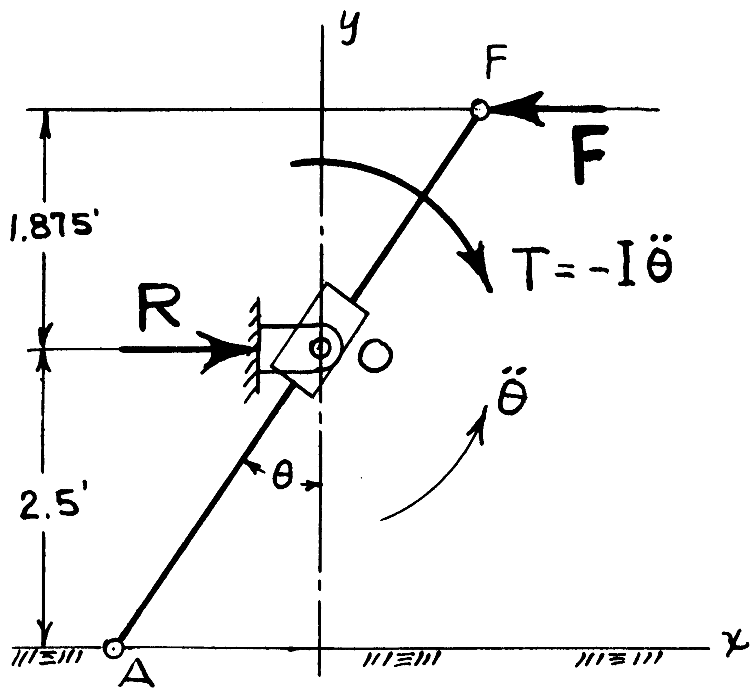


Fig. 4-6. Inertia torque on the pantograph mechanism.

So

$$t_{stride} = \frac{s}{v} = \frac{3}{29.4} = 0.102 \text{ sec}$$

Since the four events have equal times, the period is

$$\tau = (4)(0.102) = 0.408 \text{ sec}$$

For the lift and lower phases of the action we choose to use parabolic (constant acceleration) motion. The general equations for motion are

$$x, y = a + bt + ct^2 \tag{4-6}$$

where the constants a, b, and c depend upon the conditions desired at the beginning and ends of the motion. The coordinates x and y together give the position of the foot at any instant in time. The first and second time derivatives of Eq. (4-6) give the velocity and acceleration, respectively.

The coordinates to be used for this analysis are shown in Fig. 4-7. Point O, the lowest position of the lift cylinder, is the origin. The x coordinate, positive to the right, gives the displacement in the horizontal

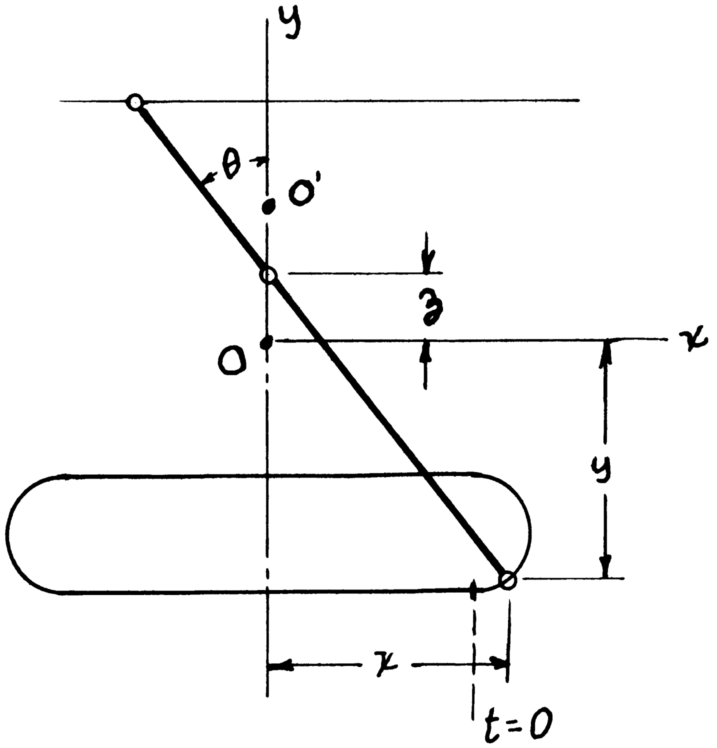


Fig. 4-7. Nomenclature.

direction. The y coordinate, positive upward, gives the vertical position of the foot. The z coordinate, positive upward, defines the vertical motion of the lift cylinder. We shall begin counting time at the end of stride, when the foot is beginning the lift action. The initial conditions for motion in the horizontal direction are

$$t = 0, \qquad x = 1.5, \qquad \dot{x} = 29.4$$

$$t = 0.102/2, \qquad \dot{x} = 0$$

The first set of conditions state that the position of the foot is 1.5 ft and its velocity is 29.4 fps when lift begins. The second set of conditions state that the velocity (in the x direction) is zero when the lift is half completed. When these conditions are substituted in Eq. (4-6) and its derivative, the constants are found to be

$$a = 1.5, \qquad b = 29.4, \qquad c = -288$$

Substituting these into Eq. (4-6) gives, for the lift

$$x = 1.5 + 29.4t - 288t^2 \tag{4-7}$$

The velocity and acceleration are

$$\dot{x} = 29.4 - 576t; \qquad \ddot{x} = -576$$

The maximum value of x occurs when t = 0.051 sec and is

$$x_{max} = 2.250$$

Before solving these equations, let us find a similar set of relations for motion in the y direction. The initial conditions are

$$t = 0, \qquad y = -2.5, \qquad \dot{y} = 0$$

$$t = 0.102/2, \qquad y = -2.125$$

When these are substituted into Eq. (4-6), and the constants evaluated, the equation becomes

$$y = -2.5 + 144t^2 \tag{4-8}$$

The velocity and acceleration are

$$\dot{y} = 288t; \qquad \ddot{y} = 288$$

Equation (4-8) is only valid for the first half of lift. For the second half, the boundary conditions are

$$t = 0.051, \qquad y = -2.125, \qquad \dot{y} = 14.70$$

and the equations become

$$y = -3.25 + 29.4t - 144t^2 \tag{4-9}$$

$$\dot{y} = 29.4 - 288t; \qquad \ddot{y} = -288$$

The relationships between z and y may be found by similar triangles. The results are

$$z = 1.071 + 0.4296 \tag{4-10}$$

$$\dot{z} = 0.429\dot{y}; \qquad \ddot{z} = 0.429\ddot{y}$$

When all of these equations are solved for various times we get the results shown in Table 4-2. Note, that at $t = 0.051$ sec, the accelerations in the vertical direction change sign instantly.

With the values of the velocity and acceleration in Table 4-2 the angle θ and the angular acceleration $\ddot{\theta}$ of the equivalent linkage may be calculated. The solution involves the Coriolis component and is best solved using a graphical approach. Since the analysis is well known, only the results will be presented here. It is also convenient to approximate the moment of inertia, graphically, while doing this. All of these results are tabulated in Table 4-3. The quantity $I\ddot{\theta}$ is the inertia torque, and F is the piston force, positive in the direction of piston motion.

The final step, in this procedure, is to prepare another table, similar to Table 4-1, for the return action. The analysis is quite similar and will not be detailed here. The results are included in Table 4-4.

The analysis is now complete and it is only necessary to put the results together to find out what is going on. Let us first tabulate all the pertinent information in a single table for easier examination. Table 4-5 gives the inertia forces on the cylinders for a single leg and for one complete cycle of operation.

If four legs, mounted at one point on the vehicle, are used, and if the action is separated in phase by one-quarter period, then the vertical inertia forces are completely balanced because when one leg is on lift, the other leg is on lower, and the two inertia forces are exactly canceled.

TABLE 4-2. KINEMATIC RELATIONS DURING LIFT

t	x	$\dot{x}$	$\ddot{x}$	y	$\dot{y}$	$\ddot{y}$	z	$\dot{z}$	$\ddot{z}$
0	1.500	29.40	-576	-2.50	0	288	0	0	123
0.01	1.765	23.64	-576	-2.49	2.88	288	0.003	1.24	123
0.02	1.973	17.88	-576	-2.44	5.76	288	0.021	2.47	123
0.03	2.123	12.10	-576	-2.37	8.64	288	0.055	3.70	123
0.04	2.215	6.40	-576	-2.27	11.52	288	0.100	4.95	123
0.051	2.250	0	-576	-2.125	14.70	0	0.160	6.30	0
0.06	2.225	- 5.20	-576	-2.00	12.10	-288	0.213	5.19	-123
0.07	2.150	-10.90	-576	-1.90	9.20	-288	0.256	3.95	-123
0.08	2.010	-16.70	-576	-1.82	6.40	-288	0.290	2.74	-123
0.09	1.810	-22.50	-576	-1.77	3.40	-288	0.311	1.46	-123
0.102	1.500	-29.40	-576	-1.75	0	-288	0.320	0	-123

TABLE 4-3. DYNAMIC RELATIONS DURING LIFT

t	x	θ, deg	I	$\ddot{\theta}$	$I\ddot{\theta}$	F
0	1.500	31.0	1.692	-230	-388	-207
0.01	1.765	35.3	1.840	-177	-326	-174
0.02	1.973	38.9	1.993	-141	-281	-152
0.03	2.123	41.0	2.065	-112	-232	-128
0.04	2.215	43.0	2.046	-100	-205	-115
0.051-	2.250	44.6	2.041	- 85	-174	-102
0.051	2.250	44.6	2.041	-156	-318	185
0.06	2.225	45.2	1.943	-168	-326	196
0.07	2.150	45.0	1.872	-180	-337	208
0.08	2.010	43.6	1.674	-216	-362	228
0.09	1.810	41.0	1.498	-266	-398	255
0.102	1.500	35.9	1.382	-337	-465	299

TABLE 4-4. DYNAMIC RELATIONS DURING RETURN

x	θ, deg	$\dot{\theta}$	$\ddot{\theta}$	I	$I\ddot{\theta}$	F
1.50	35.9	- 9.30	-126	1.240	-156	100
1.25	31.2	-10.40	-130	1.163	-151	97
1.00	25.8	-11.50	-127	0.981	-125	80
0.75	19.9	-12.55	-113	0.959	-108	70
0.50	13.6	-13.42	- 87	0.900	- 78	50
0.25	6.9	-14.00	- 47	0.858	- 40	26
0	0	-14.20	0	0.854	0	0
-0.25	6.9	-14.00	47	0.858	40	- 26
-0.50	13.6	-13.42	87	0.900	78	- 50
-0.75	19.9	-12.55	113	0.959	108	- 70
-1.00	25.8	-11.50	127	0.981	125	- 80
-1.25	31.2	-10.40	130	1.163	151	- 97
-1.50	35.9	- 9.30	126	1.240	156	-100

TABLE 4-5. INERTIA FORCES ON CYLINDERS FOR ONE LEG FOR ONE CYCLE

Time t, sec	Action	x	-y	Drive Cyl. Force	Lift Cyl. Force	Time t, sec	Action	x	-y	Drive Cyl. Force	Lift Cyl. Force
0	Stride	0	2.50	0	0	0.204	Return	0	1.75	0	0
0.0085	Stride	0.25	2.50	- 18	0	0.2125	Return	-0.25	1.75	- 26	0
0.0170	Stride	0.50	2.50	- 35	0	0.2210	Return	-0.50	1.75	- 50	0
0.0255	Stride	0.75	2.50	- 51	0	0.2295	Return	-0.75	1.75	- 70	0
0.0340	Stride	1.00	2.50	- 63	0	0.2380	Return	-1.00	1.75	- 80	0
0.0425	Stride	1.25	2.50	- 78	0	0.2465	Return	-1.25	1.75	- 97	0
0.051	Stride	1.50	2.50	- 81	0	0.2550	Return	-1.50	1.75	-100	0
0.051	Lift	1.50	2.50	-207	115	0.2550	Lower	-1.50	1.75	-299	115
0.061	Lift	1.765	2.49	-174	115	0.265	Lower	-1.810	1.77	-255	115
0.071	Lift	1.973	2.44	-152	115	0.275	Lower	-2.010	1.82	-228	115
0.081	Lift	2.123	2.37	-128	115	0.285	Lower	-2.215	1.90	-208	115
0.091	Lift	2.215	2.27	-115	115	0.295	Lower	-2.225	2.00	-196	115
0.102	Lift	2.250	2.125	-102	115	0.306	Lower	-2.250	2.125	-185	115
0.102	Lift	2.250	2.125	185	-115	0.306	Lower	-2.250	2.125	102	-115
0.112	Lift	2.225	2.00	196	-115	0.316	Lower	-2.215	2.27	115	-115
0.122	Lift	2.215	1.90	208	-115	0.326	Lower	-2.123	2.37	128	-115
0.132	Lift	2.010	1.82	228	-115	0.336	Lower	-1.973	2.44	152	-115
0.142	Lift	1.810	1.77	255	-115	0.346	Lower	-1.765	2.49	174	-115
0.153	Lift	1.50	1.75	299	-115	0.357	Lower	-1.50	2.50	207	-115
0.153	Return	1.50	1.75	100	0	0.357	Stride	-1.50	2.50	81	0
0.1615	Return	1.25	1.75	97	0	0.3655	Stride	-1.25	2.50	78	0
0.1700	Return	1.00	1.75	80	0	0.3740	Stride	-1.00	2.50	63	0
0.1785	Return	0.75	1.75	70	0	0.3825	Stride	-0.75	2.50	51	0
0.1870	Return	0.50	1.75	50	0	0.3910	Stride	-0.50	2.50	35	0
0.1955	Return	0.25	1.75	26	0	0.3995	Stride	-0.25	2.50	18	0
0.204	Return	0	1.75	0	0	0.408	Stride	0	2.50	0	0

Because the linkage is counterbalanced, there are no horizontal inertia forces.

Figure 4-8 is a graph of the drive-cylinder force plotted as a function of the piston displacement. The total piston displacement is the sum of its motion in the forward and backward directions and, consequently, a scale of 9 ft contains all four events. The area enclosed by the curve represents the work done. Since the area above the displacement axis is the same as the area below, the net work is zero. We may think of negative work as work which is returned by the legs to the power system. In a mechanical system this back-and-forth flow of energy would be handled by a flywheel. In a hydraulic system it is handled by an accumulator. The phasing of a number of legs makes it possible to smooth out this curve greatly. In order to show how this works let us consider two groups of four legs each. Each group of four legs have their action separated by one-quarter of a period. But the two groups are out of phase with each other by one-eighth of a period. Table 4-6 shows the results. For eight legs the peak force is only 38 lb or about 13 per cent of the peak force for one. It is not difficult to see that if 16 legs were employed in four groups, and if each group were out of phase by one-sixteenth

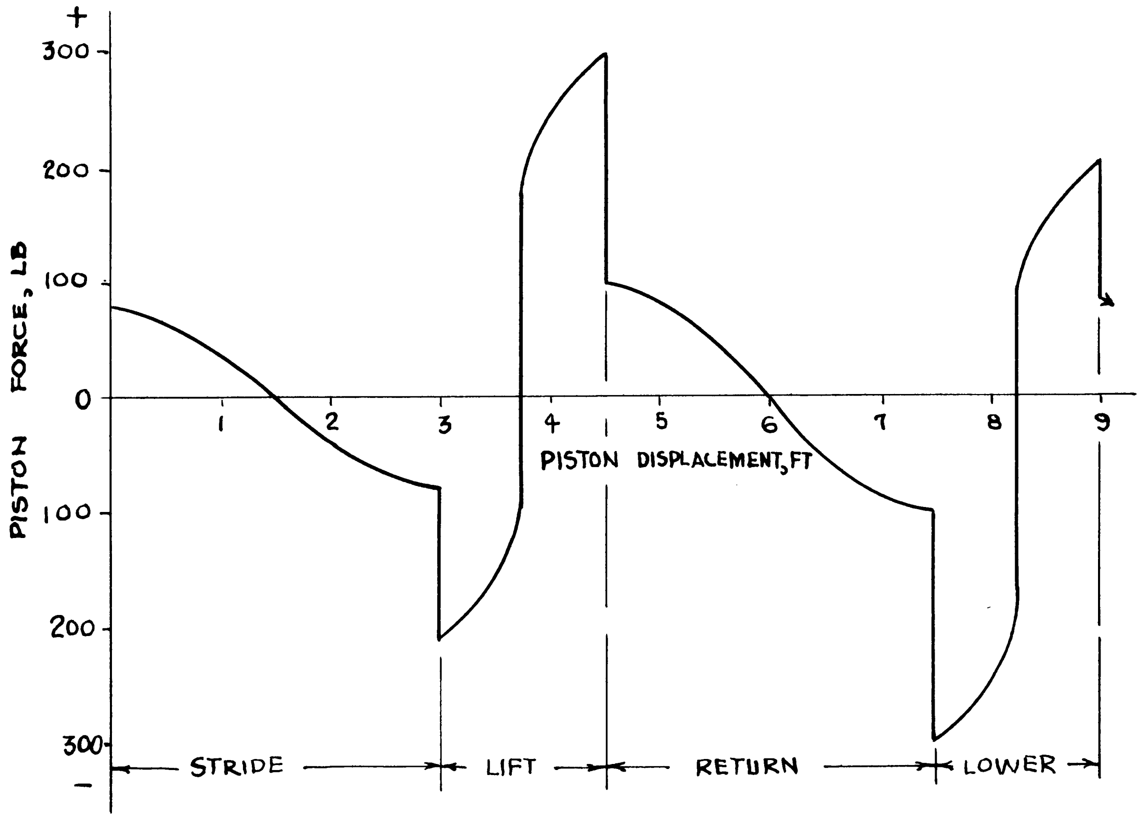


Fig. 4-8. Force-displacement diagram.

TABLE 4-6. SUMMATION OF CYLINDER FORCES

The group of four legs have their actions one-quarter period apart; the group of eight legs have their actions one-eighth period apart.

t	Drive	Lift	Return	Lower	4 Legs	8 Legs
0	81	-207	100	-299	-325	-38
0.01	72	-174	91	-255	-266	-11
0.02	60	-152	76	-228	-244	- 6
0.03	43	-128	56	-208	-235	6
0.04	24	-115	32	-196	-255	11
0.051	0	-102	0	-185	-287	38
0.051	0	185	0	102	287	-38
0.06	-24	196	- 32	115	255	-11
0.07	-43	208	- 56	128	235	- 6
0.08	-60	228	- 76	152	244	6
0.09	-72	255	- 91	174	266	11
0.102	-81	299	-100	207	325	38

TABLE 4-7. TABULATION OF TOTAL INERTIA TORQUES FOR FOUR LEGS AND FOR EIGHT LEGS

t	Drive	Lift	Return	Lower	4 Legs	8 Legs
0	152	-388	-156	465	73	-71
0.01	135	-326	-142	398	65	-51
0.02	113	-281	-118	362	76	-24
0.03	81	-232	- 87	337	100	24
0.04	45	-205	- 50	326	116	51
0.051	0	-174	0	318	144	71
0.051	0	-318	0	174	-144	-71
0.06	- 45	-326	50	205	-116	-51
0.07	- 81	-337	87	232	-100	-24
0.08	-113	-362	118	281	- 76	24
0.09	-135	-398	142	326	- 65	51
0.102	-152	-465	156	388	- 73	71

of a period, then the resulting peak would be even less. On the other hand, if two groups of four legs were in phase with two other groups, then the figures shown in the column for eight legs would be doubled. Examination of the time column shows that these figures are not exact, but they are close enough to show the degree of improvement to be expected.

Our final step in this analysis is to tabulate the inertia torques for four legs and for eight legs, phased as in the previous example. Table 4-7 shows these results. It is clear that the inertia torques are greatly reduced in magnitude.

4-3. CONTROL SYSTEM FOR PANTOGRAPH LEGS

In devising a control system for the pantograph-legged walking machine we visualize 16 legs, 4 at each corner, phased to achieve the best balance of inertia forces and torques. In particular we may desire the following;

1. Ability to control the length of stride of each gang of 4 legs.

2. Ability to control the height of step of each gang.

3. Ability to change the vehicle speed and to reverse its motion.

4. Ability to steer the vehicle by changing the speed of the legs on one side relative to the other side.

Figure 4-9 is a schematic drawing of the hydraulic system and the control system. The stride and lift for each gang of 4 legs would be controlled by ganged three-dimensional cams, four cams for stride and four for lift. The axial dimensions of these cams would control the length of stride and the height of lift, respectively. The radial shapes would initiate and control the four events of the cycle. The phase relationship of the 4 legs is dictated by the relative angular position in which the cams are fastened to the cam shaft. The cams representing two gangs of legs on one side of the vehicle could be on the same cam shaft. Then the speed of the cam shaft dictates the speed of the legs on that side.

Steering would be accomplished by speeding up the legs on one side of the vehicle. Since the legs on a single side are quite well balanced this would have a negligible effect on the inertia forces and torques transmitted to the vehicle. It would, however, increase the peaks to be supplied by the accumulator. Taking shorter steps on one side of the vehicle could not be used for steering because this does not change the speed.

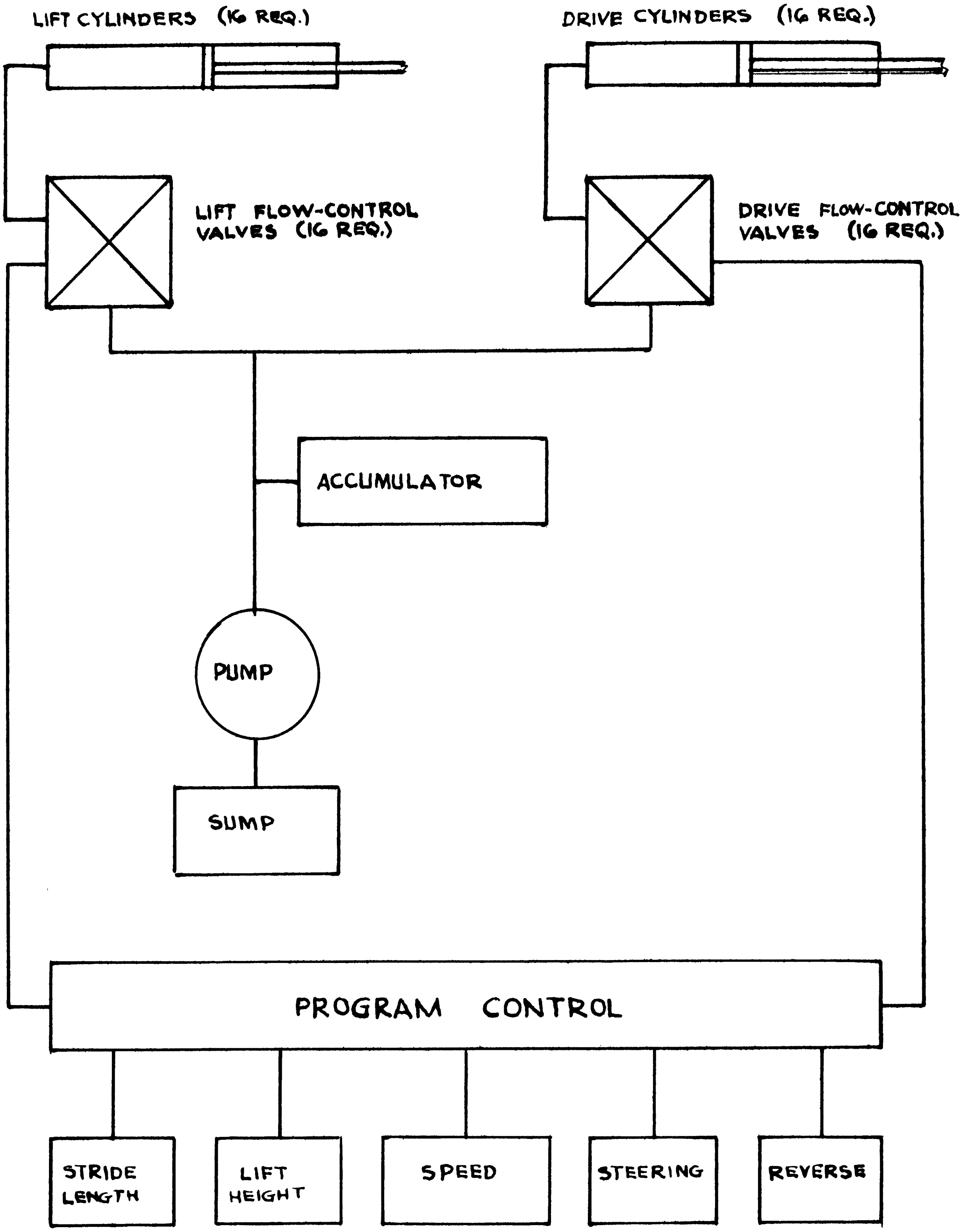


Fig. 4-9. Block diagram of control system.

The problem of how an operator can watch the action of all four feet simultaneously in order, for example, to step over a ditch, will have to be solved using an actual vehicle. It may be that, like a horse, he will instinctively know where his hind feet are!

4-4. PERFORMANCE OF THE PANTOGRAPH VEHICLE

The pantograph mechanism satisfies all of the requirements outlined in Sec. 1-2 except No. 6. It is probably impossible to get a high stride-time to return-time ratio and still get balanced inertia forces and torques no matter how the walking means is accomplished.

5. SUMMARY AND RECOMMENDATIONS

5-1. GENERAL

The disadvantages of a mechanical walking linkage, that is, one driven from a rotating power source, may be summarized as follows:

1. The driving crank must rotate at a non-uniform angular velocity in order to produce uniform vehicle velocity.

2. The length of stride and height of step are fixed.

3. The inertia forces in the horizontal and vertical directions, and the inertia torques, cannot be balanced satisfactorily.

4. The height of the body above the roadway cannot be controlled by the operator.

These disadvantages apply only to the mechanisms studied in this investigation. The possibility cannot be ruled out that a mechanical linkage could be found which does not have these disadvantages. Nevertheless, such a possibility seems very remote.

The hydraulic mechanisms, and the pantograph mechanism in particular, overcome all of these disadvantages.

In the analysis of the pantograph mechanism of the previous chapter it was shown that a walking machine should employ 16 legs in gangs of 4 each, mounted at the four corners of the vehicle in order to obtain balanced inertia forces and torques and continuously support the vehicle. It is noted that three gangs of 4 legs each, symmetrically mounted, would support the vehicle, but the inertia forces and torques would not be balanced. Using, say, six or eight gangs of 4 legs each, half on each side of the vehicle, would obtain balance. However, over rough terrain, and with stiff-legged vehicles, some of the legs would not be supporting the vehicle, since they would be "up in the air," and would be simply going along for the ride. For this reason it does not seem desirable to go to more than 16 legs.

There is a good possibility that a control system for a pantograph-legged vehicle can be found which will make it possible for the vehicle to walk in a crouch. That is, a descending foot encountering higher ground could be made to sense this fact and signal the lift cylinder to cease its downward motion at that point. This would make possible a smooth and level ride over rough terrain, and consequent higher speeds. If such a control system can be devised, then more than 16 legs could be used if found desirable. However, if the vehicle size is fixed, employing more than 16 legs may require smaller mechanisms and hence a smaller maximum step height and stride length.

We have seen that all walking machines must have shoes which permit relative motion between the shoe and the terrain, or between the shoe and the foot. Walking in muck or muddy soil would not permit relative motion between the shoe and the terrain. Consequently the shoes or feet must be designed to permit relative motion between them and their point of attachment to the walking mechanism.

5.2. SUMMARY

This investigation has shown that hydraulically operated walking machines are feasible; that the energy used by the system is zero over a complete cycle if friction is neglected; and that the speed of such machines over rough terrain is probably somewhat more than for wheeled vehicles.

The overall configuration of a control system has been described which makes it possible to control the stride length, step height, speed, and direction of motion.

The requirements for the design of the shoes or feet of such a vehicle have been defined.

In addition, methods of synthesizing the locus or path of the foot relative to the vehicle have been described, and these methods yield the velocities and accelerations directly.

The basic geometry for a walking vehicle having legs driven by a rotating crank has been described and the mechanism partially analyzed. The energy required to operate such a vehicle over a complete cycle is zero if friction is neglected. The inertia forces for such a vehicle, however, cannot be balanced. This means that such vehicles are potentially useful only for very slow speeds. Unless many legs are used, a very large flywheel is necessary if the net energy output is to be zero.

A method of synthesizing the non-circular gears required to drive the rotating-crank mechanisms has been presented. Also, the principles to be followed in defining the number of legs, and in phasing these legs to obtain minimum inertia forces and torques, have been stated.

5.3. RECOMMENDATIONS

It is conceivable that the design and construction of a walking machine could be initiated at this time. However, many unsolved problems remain, and the probability of success of such a vehicle, without further investigation and development, is small. For this reason a research and development program should be organized in order to solve the following problems:

1. The feet and shoes. Is it better to employ skis or runners, or individual feet? What is the exact geometry? What material or materials shall be used? What are the dimensions? How are the feet fastened to the linkage? How much relative motion must they permit, and how is this accomplished?

2. The linkage. What is the shape, size, dimensions, and materials? What kind of bearings shall be employed? What is size and weight of the various members in relation to the size of the vehicle to be supported by them?

3. What kind of vehicle suspension system shall be used?

4. The cylinders. How should they be connected to the linkage and to the vehicle? What size are they?

5. How shall the linkages be ganged? What are the actual mechanical details, including parts, materials, and dimensions?

6. The control system. Must it employ servos? Or can satisfactory performance be obtained by cam-driven flow-control valves? What is the exact configuration of the three-dimensional cams and how shall they be designed? What does the operator actually manipulate in controlling the vehicle—levers, wheels, push buttons, dials, or what?

7. The hydraulic system. What must be its capacity and pressure? How can the accumulator be designed? Where, on the vehicle, should the hydraulic elements be placed? What energy losses are involved?

8. What are the fuel requirements for such a vehicle and what mileage can be expected from it?

9. Can a control system be devised which will sense the operation of the feet and adjust the operation of the lift cylinders to maintain a continuously level ride? If so, what effect will bumps and rough places in the roadway now have on the balance of the legs?

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